Practice Problems from Mathematics

4 Analytic Geometry & Trigonometry

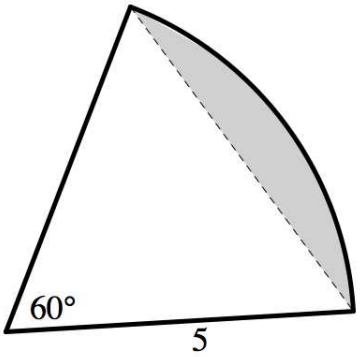
4.1 Linear Functions

1. The steel in railroad track expands when heated. For the track temperatures encountered in normal outdoor use, the length s of a piece of track is related to its temperature t by a linear equation. An experiment with a piece of track gave the following measurements: $t_1 = 65°F, s_1 = 35ft.; t_2 = 135°F,\ s_2 = 35.16ft$. Write a linear equation for s as a function of t given a point and slope or two points.

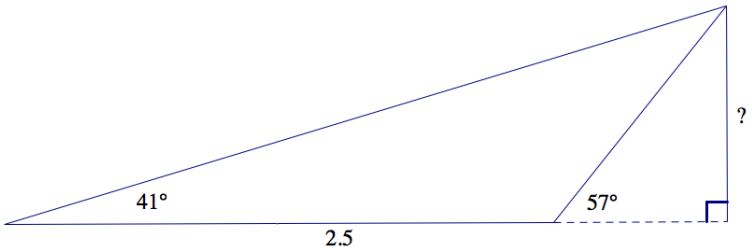
2. Find the equation for the lines parallel and perpendicular to the line $x + 2y = 3$ passing through the point $(1, 2)$.

4.2 Geometry and Conic Sections

1. 25 Find the area of the shaded region and the outlined region in the figure below. See also 4.11.7 & 4.11.15 in FE Review Manual.



2. 23 Find the missing side in the following figure.
See 4.11.14 in FE Review Manual.



3. 26-27 Determine the conic section defined by:
   (a) $2x^2 - y^2 + 4xy - 2x + 3y = 6$
   (b) $6x^2 + 3xy + 2y^2 + 17y + 2 = 0$
   (c) $x^2 - 3xy + 3y^2 + 6y = 7$
   (d) $x^2 + y^2 - x - y = 3$
   (e) $4x^2 + y^2 - 4xy + 2x - y = 0$

4. 26-27 Find the equation of the ellipse centered at $(-2, 3)$ and passing through the points $(2, 1)$ and $(-3, -3)$.

5. 26-27 Find the equation of the circle centered at $(-2, 3)$ and passing through the point $(1, -1)$.

6. 26-27 Find the equation of a parabola with:

(a) center (vertex) $(2, 4)$ and directrix $x = -3$.

(b) center $(2, 3)$ and directrix $y = -2$.

7. Eliminate the parameter in the equations $x = 4\cos t,\ y = 7\sin t$ and identify the curve traced out by the path.
   See 4.10.12 in the FE Review Manual.

5 Algebra and Linear Algebra

5.1 Logarithms and Trigonometry

1. 23 Simplify the expression or solve the equation:

   (a) $\log_{10} 40 + \log_{10}\left(\frac{5}{2}\right)$.

   (b) $\log_9 25 - \log_9 75$

   (c) $5 = 2e^{2x-1}$

   (d) $5^{3x-1} = 27$

   (e) $\ln x + \ln(x + 1) = \ln 12$

   (f) $7 \cdot 3^{-x} = 4e^{2x}$

   (g) $\log_{10} 5 = x^2 y$; find $\log_{10} 0.04$.

   (h) $\ln(3.4^z) = ?$

5.2 Matrix Arithmetic and Vectors

1. 34-35 Find the cross product, $A \times B$, of $A = 2i - 3j + 4k$ and $B = 2i + j + 3k$.

2. 34-35 For the vectors A and B above, find the dot product, $A \cdot B$, and the angle between the vectors.

3. 34-35 Find the length of the resultant of the following vectors:

   $2i + 3j + 4k$

   $-4i + 2j - k$

   $6i + 3j - 5k$

4. 34-35 Find the unit vector in the direction of $2i + 3j + k$.

5. 34-35 Find the determinant of $A = \begin{bmatrix} 3 & 2 & -2 \\ 4 & 8 & 2 \\ 3 & 3 & 1 \end{bmatrix}$.

6. 34-35 The determinant of $A = \begin{bmatrix} 3 & 10 & 4 \\ 6 & 2 & 1 \\ 1 & 3 & 4 \end{bmatrix} = -151$. Find the determinant of $A = \begin{bmatrix} 1.5 & 5 & 2 \\ 3 & 1 & 0.5 \\ 0.5 & 1.5 & 2 \end{bmatrix}$.

7. 34-35 Find the inverse of $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$.

8. 34-35 Find the cofactor matrix of $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 3 \\ 3 & 4 & 6 \end{bmatrix}$. Then, find $adj(A)$ and A^{-1}.

9. 34-35 Find the volume of the parallelopiped defined by the vectors $[4, 1, 2],\ [-1, 2, 7],\ [4, 3, 8]$. See 5.14.31 in FE Review Manual.

5.3 Sequences and Series

1. Sequences
   (a) [30] Find the sum of the finite sequence $14,\ 20,\ 26,\ 32, \ldots,\ 68$.
   (b) Find the next term in the sequence $14,\ 17,\ 20,\ 23, \ldots$.
2. More Sequences
   (a) [30] Find the sum of the finite sequence $26,\ 39,\ 58.50,\ 87.75, \ldots,\ 666.35$.
   (b) [30] Find the next term in the sequence $3,\ 21,\ 147,\ 1029, \ldots$.
3. [30] Find the first three terms in the Taylor Series of $f(x) = e^{3x}$ about $x = 0$.

5.4 Polar Coordinates and Complex Numbers

1. [23] Find the Cartesian coordinates of the point $(3, 60°)$ and the polar coordinates of $(3, 5)$ and $(-3, 5)$.
2. [23] Find the Cartesian equation for each of the following equations in polar form:
   (a) $r^2 = 4r\cos\theta$
   (b) $r = \dfrac{4}{2\cos\theta - \sin\theta}$
   (c) $r = \dfrac{3}{1 - 2\sin\theta}$
   (d) $r = 2\sin\theta + \cos\theta$
   (e) $r(2 + \sin\theta) = 1$
3. Find the x and y coordinates of the focus of the conic section with equation $r\sin^2\alpha = \cos\alpha$.
   [See 4.9.5 in FE Review Manual.]
4. [23] Complex Numbers
   (a) Find the polar form of the numbers $3 - 2j$ and $-3 + 2j$.
   (b) If $c \in \mathbb{C}$, and, in polar form, is given by $(16, 60°)$ (or $16e^{i\frac{\pi}{3}}$) then find $\sqrt[4]{c}$.
   (c) Find the product $(9 + 8j) \cdot (4 + 3j)$.
   (d) Find the rationalized form of the number $\dfrac{9 + 8j}{4 + 3j}$.

6 DIfferential & Integral Calculus

1. [30] Find $\dfrac{d}{dt}\ \cos^3\alpha t$.
2. [28-29] Find the minimum of $f(x) = 2x^3 - 6x + 4$ on the interval $[-2, 2]$. Also, find and classify all local extrema of $f(x)$.
3. [28-29] Find $\dfrac{dy}{dx}$ for $y = e^{-x^2}\sin 2x$.
   [Use calculator at a specific value and substitution.]
4. [28-29] Find $\dfrac{dy}{dx}$ for the following expressions:
   [Use partial derivatives, not implicit differentiation.]
   (a) $e^{x+y} = xy$
   (b) $5 = \cos\sqrt{xy}$

5. 28-29 Find the equation of the line tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ at the point $(1, \sqrt{2})$.
   Use partial derivatives, not implicit differentiation.

6. 28-29 Compute the following limits:
   Use L'Hôpital's Rule

   (a) $\lim_{x \to 1} \frac{x^2 + x - 2}{\sin \pi x}$

   (b) $\lim_{x \to 0} \frac{x \sin x}{\sec x - 1}$

   (c) $\lim_{x \to 0} x^2 \cot 2x$

7. 28-29 Find $\frac{\partial f}{\partial y}$ for $f(x, y) = x^3 y + \frac{3x^2}{y} + 9xy - \sin(x^2 + y)$.

8. 28-29 Find the area bounded by $y = x^2 - 3x + 4$ and $y = -x^2 + 5x - 2$.

9. 28-29 Find the area bounded by $y = \sin 2x$ and $y = \cos x$ from $x = -\frac{\pi}{2}$ to $\frac{\pi}{6}$.

10. 28-29 Find the curvature and radius of curvature of the function $f(x) = x^3 - 2x + 1$ at the point $(2, 5)$.
    Use calculator to computer necessary derivatives.

11. 35 Find the gradient vector of the function $f(x, y) = y^2 \ln \sqrt{x}$.

12. 35 For the function in the previous problem:

    (a) find the direction of the tangent line to the surface that passes through $(1, -2)$ and has maximum slope.

    (b) find that maximum slope.

    (c) find the slope of the tangent line to the surface that passes through $(1, -2)$ in the direction $3\vec{\imath} - 2\vec{\jmath}$.

    (d) find the slope of the tangent line to the surface that passes through $(1, -2)$ towards the point $(3, 0)$.

13. 35 Calculate the divergence of the vector-valued function $\vec{F}(x, y, z) = (e^x \cos z)i + (e^z \sin y)j + (xz^2)k$.

14. 35 Determine the curl of the vector-valued function $\vec{F}(x, y, z) = (x^2 z)i + (yz^2)j + (x^2 z^2)k$.

15. 35 Determine the Laplacian of the scalar function $f(x, y, z) = 3x^2 y - 2z^2 + 3$ at the point $(1, 2, 2)$.

16. 35 Determine $\frac{dy}{dx}$ for the parametric function $x(t) = 3t^2 - t,\ y(t) = 2t^2 - 6t + 7$ when $t = 1$.

17. 28-29 Evaluate the following integrals:
    Use calculator and/or substitution to check. See 7.8.7-12, 15, 18 in FE Review Manual

    (a) $\int \frac{dx}{x(2 + 3x)}$

    (b) $\int \frac{1}{9 + x^2}\, dx$

    (c) $\int \tan^2 2x\, dx$

    (d) $\int x^2 \sin x\, dx$

    (e) $\int (e^x + 2x)^2 (e^x + 2)\, dx$

18. 30 Determine the x-coordinate of the centroid of the area bounded by the function $f(x) = x^3 + 5x^2 - 3x + 5,\ y = 0,\ x = 0$, and $x = 10$.

19. 30 Find the moment of inertia about the y-axis of the area described in the previous problem.

7 Probability & Statistics

This is a separate FEE Preparation Session facilitated by Dr. Silas Bergen.

8 Differential Equations

1. 30-31 Solve the differential equation:
   Remember classifications and/or substitute each answer in, even using your calculator at a specific point, to see what works.
   (a) $4x^2y' = y^{-3} - 9y',\ y(0) = -2.$
   (b) $xy' = 3y + x^4 \cos x.$
2. 30-31 Solve the second order homogeneous differential equation $y'' + 6y' + 5y = 0$ subject to the initial conditions $y(0) = 1,\ y'(0) = 0.$
   In practice, use your brain and/or calculator to see which of the MC answers satisfies the I.C. first, then of those, which satisfies the DE.
3. 30-31 Find a particular solution to the second order non-homogeneous differential equation $y'' - y' - 2y = 10\cos x.$
4. 30-31 Solve the following differential equation using standard methods and then using Laplace Transforms: $y'' + 4y' + 4y = 0,\ y(0) = 3,\ y'(0) = 1.$
5. See problems on La Place Transforms 8.7.6-11, 13 in FE Review Manual.

9 Numerical Methods

1. 36 Use Newton's Method to approximate the solution to $x^3 - 2 = 0$. Use $a_0 = 1$ and 3 iterations.
2. 36 Use the Method of Bisection to approximate the solution to $x^3 - 2 = 0$ on $[0, 2]$ with three iterations.
3. 36 Use Euler's Method to approximate the value of $x(1)$ for the differential equation $x' = 3tx^2 + 2,\ x(0) = 0.1$ and $\Delta t = 0.5$.
4. 36 Use Simpson's Rule with $n = 8$ to approximate $\displaystyle\int_1^3 \sqrt{x^2 + 4x + 1}\, dx.$