Engineering Study Session—Mathematics Solutions

Dr. Peratt

4 Analytic Geometry & Trigonometry

4.1 Linear Functions

1. The steel in railroad track expands when heated. For the track temperatures encountered in normal outdoor use, the length s of a piece of track is related to its temperature t by a linear equation. An experiment with a piece of track gave the following measurements: $t_1 = 65°F, s_1 = 35ft.; t_2 = 135°F,\ s_2 = 35.16ft$. Write a linear equation for s as a function of t given a point and slope or two points.

   Answer: $s = 0.002286t + 34.8514$.

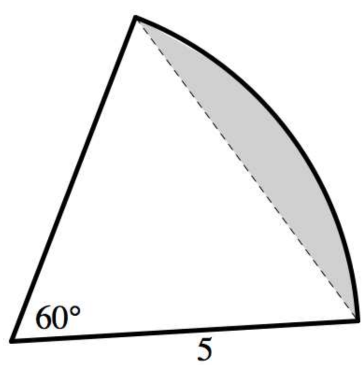
2. Find the equation for the lines parallel and perpendicular to the line $x + 2y = 3$ passing through the point $(1, 2)$.

   Answer:
   - Perpendicular: $y = 2x + 0$.
   - Parallel: $y = -\frac{1}{2}x + \frac{5}{2}$.

4.2 Geometry and Conic Sections

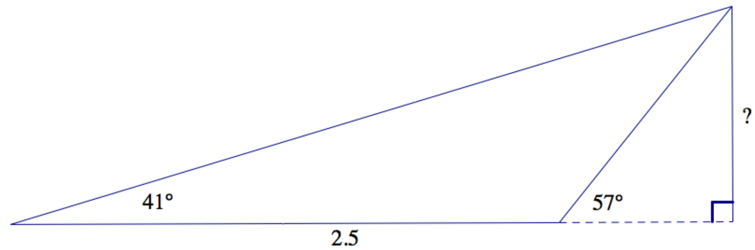
1. 25 Find the area of the shaded region and the outlined region in the figure below. See also 4.11.7 & 4.11.15 in FE Review Manual.

   Answer:
   - Shaded Region: $\frac{25}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) \approx 2.265$.
   - Outlined Region: $\frac{25\pi}{6}$.



2. 23 Find the missing side in the following figure.
   See 4.11.14 in FE Review Manual.

   Answer: We note the obtuse triangle with and 41°, side 2.5, and angles 123° and 16° (computed). If we label the angle opposite of the 41° angle x, we can use the Law of Sines to solve for x and obtain $x = 5.950$. Then, using right-triangle trigonometry on the right triangle with hypotenuse 5.950 and angle 57° we see that the side in question is $? = 5.000$.



3. [26-27] Determine the conic section defined by:

   (a) $2x^2 - y^2 + 4xy - 2x + 3y = 6$

   Answer: Using the guidelines on the top right of page 27, we see that this is a Hyperbola.

   (b) $6x^2 + 3xy + 2y^2 + 17y + 2 = 0$

   Answer: Ellipse.

   (c) $x^2 - 3xy + 3y^2 + 6y = 7$

   Answer: Ellipse.

   (d) $x^2 + y^2 - x - y = 3$

   Answer: Ellipse.

   (e) $4x^2 + y^2 - 4xy + 2x - y = 0$

   Answer: Parabola.

4. [26-27] Find the equation of the ellipse centered at $(-2, 3)$ and passing through the points $(2, 1)$ and $(-3, -3)$.

   Answer: $\dfrac{(x-2)^2}{5^2} + \dfrac{(y-(-3))^2}{4^2} = 1.$

5. [26-27] Find the equation of the circle centered at $(-2, 3)$ and passing through the point $(1, -1)$.

   Answer: $(x + 2)^2 + (y - 3)^2 = 25.$

6. [26-27] Find the equation of a parabola with:

   (a) center (vertex) $(2, 4)$ and directrix $x = -3$.

   Answer: $(y - 4)^2 = 2 \cdot 6 \cdot (x - 2)$, where $p = 6$.

   (b) center $(2, 3)$ and directrix $y = -2$.

   Answer: $(x - 2)^2 = 2 \cdot 4 \cdot (y - 3)$.

7. Eliminate the parameter in the equations $x = 4\cos t$, $y = 7\sin t$ and identify the curve traced out by the path.
   [See 4.10.12 in the FE Review Manual.]

   Answer: Solving the equations for sine and cosine we get $\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{7}$. Therefore, invoking the identity $\sin^2 t + \cos^2 t = 1$, we obtain $\dfrac{y^2}{7^2} + \dfrac{x^2}{4^2} = 1$ and see that this is an ellipse centered at the origin with semi-major axis of 7 in the y-direction and semi-minor axis of 4 in the x-direction.

5 Algebra and Linear Algebra

5.1 Logarithms and Trigonometry

1. [23] Simplify the expression or solve the equation:

   (a) $\log_{10} 40 + \log_{10}\left(\frac{5}{2}\right)$.

   Answer: 2.

(b) $\log_9 25 - \log_9 75$

Answer: $-\frac{1}{2}$.

(c) $5 = 2e^{2x-1}$

Answer: $\dfrac{1+\ln 2.5}{2} \approx 0.958\ldots.$

(d) $5^{3x-1} = 27$

Answer: $\dfrac{\frac{\ln 27}{\ln 5}+1}{3} \approx 1.01594\ldots.$

(e) $\ln x + \ln(x+1) = \ln 12$

Answer: $x = -4, 3$.

(f) $7 \cdot 3^{-x} = 4e^{2x}$

Answer: $x = \dfrac{\ln\left(\frac{7}{4}\right)}{2+\ln 3} \approx 0.1806\ldots.$

(g) $\log_{10} 5 = x^2 y$; find $\log_{10} 0.04$.

Answer: $\log_{10} 0.04 = \log_{10} 5^{-2} = -2 \cdot \log_{10} 5 = -2x^2 y$.

(h) $\boxed{\ln(3.4^z) = ?}$

Answer: $z \cdot \ln 3.4$.

5.2 Matrix Arithmetic and Vectors

1. $\boxed{\text{34-35}}$ Find the cross product, $A \times B$, of $A = 2i - 3j + 4k$ and $B = 2i + j + 3k$.

   Answer: $-13\vec{\imath} + 2\vec{\jmath} + 8\vec{k}$.

2. $\boxed{\text{34-35}}$ For the vectors A and B above, find the dot product, $A \cdot B$, and the angle between the vectors.

   Answer: $\cos\theta = A \cdot B|A||B| = 0.645$, so $\theta = \arccos(0.645) \approx 0.8695 \approx 49.8°$.

3. $\boxed{\text{34-35}}$ Find the length of the resultant of the following vectors:

   $2i + 3j + 4k$

   $-4i + 2j - k$

   $6i + 3j - 5k$

   Answer: $\sqrt{84} \approx 9.165$.

4. $\boxed{\text{34-35}}$ Find the unit vector in the direction of $2i + 3j + k$.

   Answer: $\frac{2}{\sqrt{14}}\vec{\imath} + \frac{3}{\sqrt{14}}\vec{\jmath} + \frac{1}{\sqrt{14}}\vec{k}$.

5. $\boxed{\text{34-35}}$ Find the determinant of $A = \begin{bmatrix} 3 & 2 & -2 \\ 4 & 8 & 2 \\ 3 & 3 & 1 \end{bmatrix}$.

   Answer: 34.

6. $\boxed{\text{34-35}}$ The determinant of $A = \begin{bmatrix} 3 & 10 & 4 \\ 6 & 2 & 1 \\ 1 & 3 & 4 \end{bmatrix} = -151$. Find the determinant of $A = \begin{bmatrix} 1.5 & 5 & 2 \\ 3 & 1 & 0.5 \\ 0.5 & 1.5 & 2 \end{bmatrix}$.

   Answer: -75.5, or half of the original, since the new matrix is simply 0.5 multiplied by the original matrix.

7. $\boxed{\text{34-35}}$ Find the inverse of $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$.

   Answer:

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{3}{4} \end{pmatrix}$$

8. 34-35 Find the cofactor matrix of $A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 3 \\ 3 & 4 & 6 \end{bmatrix}$. Then, find $adj(A)$ and A^{-1}.

Answer: The cofactor Matrix is

$$\begin{pmatrix} -6 & -15 & 13 \\ -14 & 9 & 1 \\ 8 & -2 & -10 \end{pmatrix},$$

and

$$adj(A) = \begin{pmatrix} -6 & -14 & -8 \\ -15 & 9 & -2 \\ 13 & 1 & -10 \end{pmatrix}.$$

Therefore,

$$A^{-1} = \frac{adj(A)}{|A|} = \begin{pmatrix} \frac{3}{22} & \frac{7}{22} & -\frac{2}{11} \\ \frac{15}{44} & -\frac{9}{44} & \frac{1}{22} \\ -\frac{13}{44} & -\frac{1}{44} & \frac{5}{22} \end{pmatrix}.$$

9. 34-35 Find the volume of the parallelopiped defined by the vectors $[4,1,2]$, $[-1,2,7]$, $[4,3,8]$. See 5.14.31 in FE Review Manual.

Answer: The absolute value of the triple scalar product, $A \cdot (B \times C)$ gives the volume of the parallelopiped, regardless of how we choose vectors A, B, and C. Therefore, we compute

$$|[4,1,2] \cdot ([-1,2,7] \times [4,3,8])| = |[4,1,2] \cdot [-5,36,-11]| = |-6| = 6.$$

5.3 Sequences and Series

1. Sequences

   (a) 30 Find the sum of the finite sequence $14, \ 20, \ 26, \ 32, \ldots, \ 68$.

   Answer: We subtract consecutive terms to see that it is an arithmetic sequence with constant difference 6 so, applying the formula on page 30, we see that the sum is 410.

   (b) Find the next term in the sequence $14, \ 17, \ 20, \ 23, \ldots$.

   Answer: 26.

2. More Sequences

   (a) 30 Find the sum of the finite sequence $26, \ 39, \ 58.50, \ 87.75, \ldots, \ 666.35$.

   Answer: We divide consecutive terms to see that it is a geometric sequence with constant ratio 1.5 so, applying the formula on page 30, we see that the sum is 1947.05.

   (b) 30 Find the next term in the sequence $3, \ 21, \ 147, \ 1029, \ldots$.

   Answer: 7203.

3. 30 Find the first three terms in the Taylor Series of $f(x) = e^{3x}$ about $x = 0$.

   Answer: $1 + 3x + \frac{9}{2}x^2 + \ldots$.

5.4 Polar Coordinates and Complex Numbers

1. 23 Find the Cartesian coordinates of the point $(3, 60°)$ and the polar coordinates of $(3,5)$ and $(-3,5)$.

   Answer: The Cartesian coordinates of $(3, 60°)$ are $(1.5, 2.598)$. The polar coordinates of $(3,5)$ are $(6, 1.030)$, and for $(-3,5)$ are $(6, 4.172)$.

2. 23 Find the Cartesian equation for each of the following equations in polar form:

(a) $r^2 = 4r\cos\theta$

Answer: $x^2 - 4x + y^2 = 0$.

(b) $r = \dfrac{4}{2\cos\theta - \sin\theta}$

Answer: $y = 2x - 4$

(c) $r = \dfrac{3}{1 - 2\sin\theta}$

Answer: $x^2 - 3y^2 - 12y - 9 = 0$.

(d) $r = 2\sin\theta + \cos\theta$

Answer: $x^2 - x + y^2 - 2y = 0$.

(e) $r(2 + \sin\theta) = 1$

Answer: $4x^2 + 3y^2 + 2y - 1 = 0$.

3. Find the x and y coordinates of the focus of the conic section with equation $r\sin^2\alpha = \cos\alpha$.
See 4.9.5 in FE Review Manual.

Answer: We first convert this to Cartesian coordinates by multiplying each side by r and using the conversion formulas on the top right of p. 23 to get $r^2\sin^2\alpha = r\cos\alpha \to y^2 = x$, or $x = y^2$. Therefore, rewriting the equation in the form of the one given on the top of page 26 gives $(y-0)^2 = 2\cdot\frac{1}{2}\cdot(x-0)$, so that the coordinates of the focus are $\left(\frac{p}{2}, 0\right) = \left(\frac{1}{4}, 0\right)$.

4. 23 Complex Numbers

(a) Find the polar form of the numbers $3 - 2j$ and $-3 + 2j$.

Answer: $r = \sqrt{13}$ and $\theta = -0.588$, so the polar form is $\sqrt{13}e^{-0.588j}$.

(b) If $c \in \mathbb{C}$, and, in polar form, is given by $(16, 60°)$ (or $16e^{i\frac{\pi}{3}}$) then find $\sqrt[4]{c}$.

Answer: $\sqrt[4]{c} = \left(\sqrt[4]{16}, \dfrac{60°}{4}\right) = (2, 15°)$.

(c) Find the product $(9 + 8j)\cdot(4 + 3j)$.

Answer: $(9+8j)(4+3j) = 36 + 27j + 32j + 24j^2 = 36 + 59j - 24 = 14 + 59j$.

(d) Find the rationalized form of the number $\dfrac{9+8j}{4+3j}$.

Answer: $\dfrac{9+8j}{4+3j}\cdot\dfrac{4-3j}{4-3j} = \dfrac{60+5j}{25} = \dfrac{12}{5} + \dfrac{1}{5}j$.

6 DIfferential & Integral Calculus

1. 30 Find $\dfrac{d}{dt}\cos^3\alpha t$.

Answer: $\dfrac{d}{dt}\cos^3\alpha t = \dfrac{d}{dt}(\cos\alpha)^3 = 3(\cos\alpha)^2\cdot(-\sin\alpha)$.

2. 28-29 Find the minimum of $f(x) = 2x^3 - 6x + 4$ on the interval $[-2, 2]$. Also, find and classify all local extrema of $f(x)$.

Answer: $x = -1$ is a local max and $x = 1$ is a local min. The global min is 0 and it occurs at $x = -2$ and $x = 1$.

3. 28-29 Find $\dfrac{dy}{dx}$ for $y = e^{-x^2}\sin 2x$.
Use calculator at a specific value and substitution.

Answer: $y' = e^{-x^2}\cdot 2\cos 2x - 2xe^{-x^2}\cdot\sin 2x$.

4. 28-29 Find $\dfrac{dy}{dx}$ for the following expressions:

Use partial derivatives, not implicit differentiation.

(a) $e^{x+y} = xy$

Answer: Rewrite the equation as $e^{x+y} - xy = 0$, and define $f(x,y) = e^{x+y} - xy$. Then, $y' = -\dfrac{f_x}{f_y} = -\dfrac{e^{x+y} - y}{e^{x+y} - x}$.

(b) $5 = \cos\sqrt{xy}$

Answer: Similarly, $y' = -\dfrac{f_x}{f_y} = \dfrac{-\sin\sqrt{xy}\cdot\frac{1}{2\sqrt{xy}}\cdot y}{-\sin\sqrt{xy}\cdot\frac{1}{2\sqrt{xy}}\cdot x} = -\dfrac{y}{x}$.

5. 28-29 Find the equation of the line tangent to the ellipse $\dfrac{x^2}{2} + \dfrac{y^2}{4} = 1$ at the point $(1, \sqrt{2})$.

Use partial derivatives, not implicit differentiation.

Answer: $y - 2 = -\frac{2}{\sqrt{2}}\cdot(x-1)$.

6. 28-29 Compute the following limits:

Use L'Hôpital's Rule

(a) $\displaystyle\lim_{x\to 1}\frac{x^2+x-2}{\sin\pi x}$

Answer: $-\frac{3}{\pi}$.

(b) $\displaystyle\lim_{x\to 0}\frac{x\sin x}{\sec x - 1}$

Answer: 2.

(c) $\displaystyle\lim_{x\to 0} x^2\cot 2x$

Answer: 0.

7. 28-29 Find $\dfrac{\partial f}{\partial y}$ for $f(x,y) = x^3y + \dfrac{3x^2}{y} + 9xy - \sin(x^2+y)$.

Answer: $x^3 + 3x^2\cdot\dfrac{-1}{y^2} + 9x - \cos(x^2+y^2)\cdot 1$.

8. 28-29 Find the area bounded by $y = x^2 - 3x + 4$ and $y = -x^2 + 5x - 2$.

Answer: You can find the limits of integration using the numerical solver on the *TI-36X Pro*, $x = 1$ and $x = 3$. Hence, the area is given by $\left|\displaystyle\int_1^3 ((x^2-3x+4) - (-x^2+5x-2))\,dx\right| = \left|-\dfrac{8}{3}\right| = \dfrac{8}{3}$.

9. 28-29 Find the area bounded by $y = \sin 2x$ and $y = \cos x$ from $x = -\frac{\pi}{2}$ to $\frac{\pi}{6}$.

Answer: $\left|\displaystyle\int_{-\pi/2}^{\pi/6} (\sin(2x) - \cos(x))\,dx\right| = 2.25$.

10. 28-29 Find the curvature and radius of curvature of the function $f(x) = x^3 - 2x + 1$ at the point $(2,5)$.

Use calculator to computer necessary derivatives.

Answer: Use the formula on page 28 and your *TI-36X Pro* to complete the differentiation to obtain $\kappa \approx 0.118$ and $R = \frac{1}{\kappa} \approx 84.587$.

11. 35 Find the gradient vector of the function $f(x,y) = y^2\ln\sqrt{x}$.

Answer: $\overrightarrow{\nabla f} = \left(\frac{1}{2}y^2\cdot\frac{1}{x}\right)\vec{\imath} + (2y\ln\sqrt{x})\vec{\jmath}$.

12. 35 For the function in the previous problem:

(a) find the direction of the tangent line to the surface that passes through $(1, -2)$ and has maximum slope.

Answer: $\overrightarrow{\nabla f}(1, 2) = 2\vec{\imath} + 0\vec{\jmath}$.

(b) find that maximum slope.

Answer: $\left|\overrightarrow{\nabla f}(1, 2)\right| = 2$.

(c) find the slope of the tangent line to the surface that passes through $(1, -2)$ in the direction $3\vec{\imath} - 2\vec{\jmath}$.

Answer: This is another way of asking us for the slope of the surface at the point $(1, -2)$ in the direction of $3\vec{\imath} - 2\vec{\jmath}$. To find this, we dot the gradient with the *unit* vector in the appropriate direction: $(2\vec{\imath} + 0\vec{\jmath}) \cdot \left(\frac{1}{\sqrt{13}}(3\vec{\imath} - 2\vec{\jmath})\right) = \frac{6}{\sqrt{13}}$.

(d) find the slope of the tangent line to the surface that passes through $(1, -2)$ towards the point $(3, 0)$.

Answer: Similarly, this asks for the slope of the surface at the point $(1, -2)$ in the direction of the vector that points from $(1, -2)$ to $(3, 0)$, or $2\vec{\imath} + 2\vec{\jmath}$. This is $\sqrt{2}$.

13. [35] Calculate the divergence of the vector-valued function $\vec{F}(x, y, z) = (e^x \cos z)i + (e^z \sin y)j + (xz^2)k$.

Answer: $(e^x \cos z)\vec{\imath} + (e^z \cos y)\vec{\jmath} + (2xz)\vec{k}$.

14. [35] Determine the curl of the vector-valued function $\vec{F}(x, y, z) = (x^2 z)i + (yz^2)j + (x^2 z^2)k$.

Answer: $(0 - 2yz)\vec{\imath} + (-2xz^2 + x^2)\vec{\jmath} + (0 - 0)\vec{k}$.

15. [35] Determine the Laplacian of the scalar function $f(x, y, z) = 3x^2 y - 2z^2 + 3$ at the point $(1, 2, 2)$.

Answer: $6y + 6x - 4$.

16. [35] Determine $\dfrac{dy}{dx}$ for the parametric function $x(t) = 3t^2 - t,\ y(t) = 2t^2 - 6t + 7$ when $t = 1$.

Answer: $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \left.\dfrac{4t - 6}{6t - 1}\right|_{t=1} = \dfrac{-2}{-5} = \dfrac{2}{5}$.

17. [28-29] Evaluate the following integrals:

[Use calculator and/or substitution to check. See 7.8.7-12, 15, 18 in FE Review Manual]

(a) $\displaystyle\int \frac{dx}{x(2 + 3x)}$

(b) $\displaystyle\int \frac{1}{9 + x^2}\, dx$

(c) $\displaystyle\int \tan^2 2x\ dx$

(d) $\displaystyle\int x^2 \sin x\ dx$

(e) $\displaystyle\int (e^x + 2x)^2 (e^x + 2)\ dx$

18. [30] Determine the x-coordinate of the centroid of the area bounded by the function $f(x) = x^3 + 5x^2 - 3x + 5,\ y = 0,\ x = 0$, and $x = 10$.

Answer: Use the formulas on the top left of page 30 and your *TI-36X Pro* to compute the relevant integrals: $x_c = \dfrac{\int x\ dA}{A} = \dfrac{\int x \cdot f(x)\ dx}{A} = \dfrac{\int_0^{10} x(x^3 + 5x^2 - 3x + 5)\ dx}{\int_0^{10} (x^3 + 5x^2 - 3x + 5)\ dx} = \dfrac{31750}{12,200/3} = \dfrac{1904}{244} \approx$ 7.807.

19. [30] Find the moment of inertia about the y-axis of the area described in the previous problem.

Answer: $M_y = x_c \cdot A = 31,750$.

7 Probability & Statistics

This is a separate FEE Preparation Session facilitated by Dr. Silas Bergen.

8 Differential Equations

1. 30-31 Solve the differential equation:
   Remember classifications and/or substitute each answer in, even using your calculator at a specific point, to see what works.
   (a) $4x^2y' = y^{-3} - 9y',\ y(0) = -2$.

   Answer: The is separable and becomes $y^3\ dy = \dfrac{1}{4x^2+9}\ dx$. Integrating both sides of the equation yields $\frac{1}{4}y^4 = \frac{1}{6}\text{arctan}\left(\frac{2x}{3}\right) + C \to y = -\sqrt{\frac{2}{3}\text{arctan}\left(\frac{2x}{3}\right) + C}$. We choose the negative in front of the square root in order to satisfy the initial condition, which calls for an initial y value of -2.

   (b) $xy' = 3y + x^4 \cos x$.

   Answer: This is a linear equation and, in standard form, is $y' - \frac{3}{x}y = x^3 \cos x$. Hence, the integrating factor is $I = e^{\int -\frac{3}{x}\ dx} = e^{-3\ln x} = e^{lnx^{-3}} = x^{-3}$. Multiplying both sides of the equation by this gives $x^{-3}y' - 3x^{-4}y = \cos x$, so that $\left(x^{-3}y\right)' = \cos x$ and $x^{-3}y = \sin x + C$. Hence, the solution is $y = x^3 \sin x + cx^3$.

2. 30-31 Solve the second order homogeneous differential equation $y'' + 6y' + 5y = 0$ subject to the initial conditions $y(0) = 1,\ y'(0) = 0$.
   In practice, use your brain and/or calculator to see which of the MC answers satisfies the I.C. first, then of those, which satisfies the DE.

   Answer: Remember that the characteristic equation is $r^2+6r+5 = 0$, so that $r = -1,\ -5$. Hence, the general solution is $y = Ae^{-1t} + Be^{-5t}$. Then, substitute in the I.C. and solve the resulting equations to find the values of A and B.

3. 30-31 Find a particular solution to the second order non-homogeneous differential equation $y'' - y' - 2y = 10 \cos x$.

   Answer: Don't do this outright. Use the method in the box above.

4. 30-31 Solve the following differential equation using standard methods and then using Laplace Transforms: $y'' + 4y' + 4y = 0,\ y(0) = 3,\ y'(0) = 1$.

   Answer: It does not seem that the FEE asks you to solve DEs using the method of La Place directly.

5. See problems on La Place Transforms 8.7.6-11, 13 in FE Review Manual.

9 Numerical Methods

1. 36 Use Newton's Method to approximate the solution to $x^3 - 2 = 0$. Use $a_0 = 1$ and 3 iterations.

   Answer: The method uses the the formula on page 36: $a_{j+1} = a_j - \dfrac{f(a_j)}{f'(a_j)} = a_j - \dfrac{a_j^3 - 2}{3a_j^2}$. Beginning with $a_0 = 1$, we find that $a_1 = 1.333\ldots,\ a_2 = 1.26389\ldots$, and $a_3 = 1.25993\ldots$.

2. 36 Use the Method of Bisection to approximate the solution to $x^3 - 2 = 0$ on $[0, 2]$ with three iterations.

Answer: The function $f(x) = x^3 - 2$ is negative at $x = 0$ and $x = 1$ but positive at $x = 2$, hence the root must be between $x = 1$ and $x = 2$. Repeating this process on the interval $[1, 2]$, we see that the function is negative at $x = 1$ but positive at both $x = 1.5$ and $x = 2$. Hence, the root must be in the interval $[1, 1.5]$. Repeating this process, we find that the root must be in the interval $[1.25, 1.5]$. Hence, the root is $1.25\ldots$.

3. $\boxed{36}$ Use Euler's Method to approximate the value of $x(1)$ for the differential equation $x' = 3tx^2+2$, $x(0) = 0.1$ and $\Delta t = 0.5$.

   Answer: Euler's Method uses the formula $x_{j+1} = x_j + f(x_j, t_j)\cdot\Delta t$, where $f(x,t) = 3tx^2+2$, $x_0 = 0.1$, and $t_0 = 0$. Thus, iterating this gives $x(1) \approx x_2 = 3.0075$.

4. $\boxed{36}$ Use Simpson's Rule with $n = 8$ to approximate $\displaystyle\int_1^3 \sqrt{x^2+4x+1}\,dx$.

   Answer: We note that $\Delta x = \frac{3-1}{8} = 0.25$ and use the function $f(x) = \sqrt{x^2+4x+1}$ to create a table of values:

x	y
$x_0 = 1$	$y_0 = f(1) = \sqrt{6}$
$x_1 = 1.25$	$y_1 = f(1.25) = 2.75$
$x_2 = 1.50$	$y_2 = f(1.50) = 3.04138\ldots$
$\vdots$	$\vdots$
$x_8 = 3$	$y_8 = f(3) = 4.69042\ldots$

   Then, applying Simpson's Rule formula, we get the following approximation for our integral:

$$\frac{\Delta x}{3}\cdot(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 + 2y_6 + 4y_7 + y_8) \approx \frac{0.25}{3}\cdot 86.2605\cdots = 7.188\ldots.$$