

Annotation Key

- = substitute / use calculator
- = minimal knowledge
- = use formula reference
- = more knowledge / thought required
- × = time investment not worth it

The radius is

$$r = \sqrt{25} = 5$$

Answer is D.

Problem 5

What are the solutions to the following equation?

$$x^2 - x - 12 = 0$$

- (A) $x_1 = 1; x_2 = 12$
- (B) $x_1 = 4; x_2 = -3$
- (C) $x_1 = -1; x_2 = 4$
- (D) $x_1 = 6; x_2 = -2$

Solution

There are two ways to solve the equation. The first method is to factor the equation.

$$x^2 - x - 12 = (x + 3)(x - 4) = 0$$

$$x = -3 \text{ or } x = 4$$

The second method is to use the quadratic equation.

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - (4)(1)(-12)}}{(2)(1)}$$

$$= \frac{1 \pm 7}{2}$$

$$x_1 = 4$$

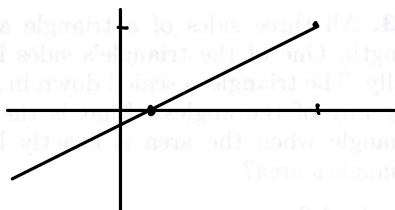
$$x_2 = -3$$

Answer is B.

FE-STYLE EXAM PROBLEMS

1. What is the length of the line with slope $4/3$, from the point $(6, 4)$ to the y -axis?

- (A) 10
- (B) 25
- (C) 50
- (D) 75



2. A line goes through the point $(4, -6)$ and is perpendicular to the line $y = 4x + 10$. What is the equation of the line?

- (A) $y = mx - 20$
- (B) $y = -\frac{1}{4}x - 5$
- (C) $y = \frac{1}{5}x + 5$
- (D) $y = \frac{1}{4}x + 5$

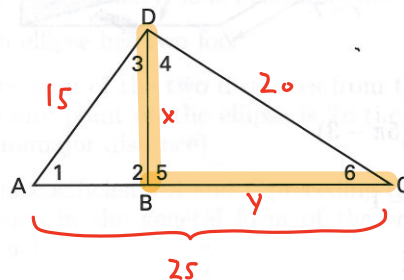
$m = -\frac{1}{4}$

3. Which of the following expressions is equivalent to the expression $\csc \theta \cos^3 \theta \tan \theta$?

- (A) $\sin \theta$
- (B) $\cos \theta$
- (C) $1 - \sin^2 \theta$
- (D) $1 + \sin^2 \theta$

Check with $\theta = 1.7$

4. In the following illustration, angles 2 and 5 are 90° , $AD = 15$, $DC = 20$, and $AC = 25$. What are the lengths BC and BD , respectively?



- (A) 12, 16
- (B) 16, 12
- (C) 13, 17
- (D) 18, 13

$$\frac{25}{20} = \frac{15}{x}$$

$$\frac{25}{20} = \frac{20}{y}$$

$$25x = 300$$

$$x = 12$$

$$y = 16$$

5. What are the x - and y -coordinates of the focus of the conic section described by the following equation? (Angle α corresponds to a right triangle with adjacent side x , opposite side y , and hypotenuse r .)

- (A) $(-\frac{1}{2}, 0)$
- (B) $(0, 0)$
- (C) $(0, \frac{\pi}{2})$
- (D) $(\frac{1}{4}, 0)$

$$r \sin^2 \alpha = \cos \alpha$$

$$r^2 \sin^2 \alpha = r \cos \alpha$$

$$(r \sin \alpha)^2 = r \cos \alpha$$

$$y^2 = x$$

Use formulas for conic sections

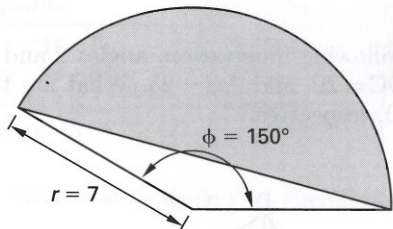
Mathematics

For the following problems, use the NCEES Handbook as your only reference.

6. What is the equation of the ellipse with center at $(0,0)$ that passes through the points $(2,0)$, $(0,3)$, and $(-2,0)$?

- (A) $\frac{x^2}{9} - \frac{y^2}{4} = 1$
- (B) $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- (C) $\frac{x^2}{9} + \frac{y^2}{4} = 1$
- (D) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

7. What is the area of the portion of the circle shown?



- (A) $\frac{49}{12}(5\pi - 3)$
- (B) $\frac{5\pi}{6} - 1$
- (C) $\frac{50\pi}{3}$
- (D) $49\pi - \sqrt{3}$

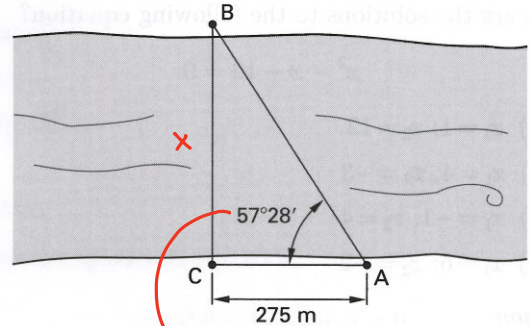
8. What is the equation of the circle passing through the (x,y) points $(0,0)$, $(0,4)$, and $(-4,0)$?

- (A) $(x-2)^2 + (y-2)^2 = \sqrt{8}$
- (B) $(x-2)^2 + (y-2)^2 = 8$
- (C) $(x+2)^2 + (y-2)^2 = 8$
- (D) $(x+2)^2 + (y+2)^2 = \sqrt{8}$

9. Which of the following equations describes a circle with center at $(2,3)$ and passing through the point $(-3,-4)$?

- (A) $(x+3)^2 + (y+4)^2 = 85$
- (B) $(x+3)^2 + (y+2)^2 = \sqrt{74}$
- (C) $(x-3)^2 + (y-2)^2 = 74$
- (D) $(x-2)^2 + (y-3)^2 = 74$

10. To find the width of a river, a surveyor sets up a transit at point C on one river bank and sights directly across to point B on the other bank. The surveyor then walks along the bank for a distance of 275 m to point A. The angle CAB is $57^\circ 28'$. What is the approximate width of the river?



- (A) 148 m
- (B) 231 m
- (C) 326 m
- (D) 431 m

$57 + \frac{28}{60} = 57.47^\circ$
 $\tan 57.47 = \frac{x}{275}$

11. Which of the following expressions is equivalent to $\sin 2\theta$?

- (A) $2 \sin \theta \cos \theta$
- (B) $\cos^2 \theta - \sin^2 \theta$
- (C) $\sin \theta \cos \theta$
- (D) $\frac{1 - \cos 2\theta}{2}$

Check with $\theta = 1.7$

12. The x - and y -coordinates of a particle moving in the x - y plane are $x = 8 \sin t$ and $y = 6 \cos t$. Which of the following equations describes the path of the particle?

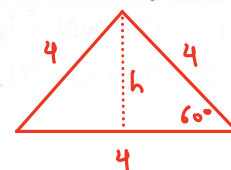
- (A) $36x^2 + 64y^2 = 2304$
- (B) $6x^2 + 8y^2 = 10$
- (C) $64x^2 + 36y^2 = 2304$
- (D) $64x^2 - 36y^2 = 2304$

$x = 7.933$
 $y = -0.733$

$36x^2 + 64y^2 = 2304 \checkmark$

13. All three sides of a triangle are initially 4 m in length. One of the triangle's sides is oriented horizontally. The triangle is scaled down in size without changing any of the angles. What is the new height of the triangle when the area is exactly half of the original triangle's area?

- (A) 1.2 m
- (B) 1.5 m
- (C) 1.7 m
- (D) 2.5 m



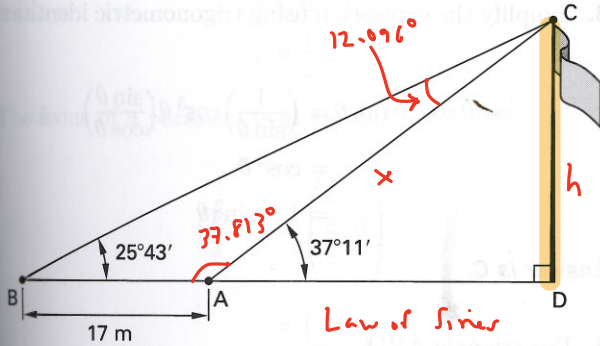
$h = 4 \sin 60^\circ$
 $= 2\sqrt{3} \approx 3.464$

$A = \frac{1}{2} b \cdot h$

To make A one-half of original, cut h

in half: $1.732 = \text{new } h$

14. The vertical angle to the top of a flagpole from point A on the ground is observed to be $37^\circ 11'$. The observer walks 17 m directly away from point A and the flagpole to point B and finds the new angle to be $25^\circ 43'$. What is the approximate height of the flagpole?



Law of Sines

$$\frac{\sin 12.096^\circ}{17} = \frac{\sin 25.717^\circ}{x}$$

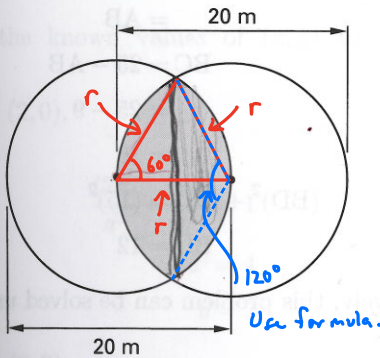
$$x = 35.203$$

$$\sin 37.813^\circ = \frac{h}{35.203}$$

$$h = 21.275$$

- (A) 10 m
 (B) 22 m
 (C) 82 m
 (D) 300 m

15. Two 20 m diameter circles are placed so that the circumference of each just touches the center of the other. What is the area common to each circle?



- (A) 61.4 m^2
 (B) 105 m^2
 (C) 123 m^2
 (D) 166 m^2

16. What conic section is described by the following equation?

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$4x^2 - y^2 + 8x + 4y = 15$$

- (A) circle $A = 4$
 (B) ellipse $B = 0$
 (C) parabola $C = -1$
 (D) hyperbola $D = 8$
 $E = 4$
 $F = -15$
 $B^2 - 4AC = 16 > 0$
 hyperbola

17. What is the equation of a parabola with a vertex at (4, 8) and a directrix at $y = 5$?

- (A) $(x - 8)^2 = 12(y - 4)$
 (B) $(x - 4)^2 = 12(y - 8)$
 (C) $(x - 4)^2 = 6(y - 8)$
 (D) $(y - 8)^2 = 12(x - 4)$

18. Which of the following statements is FALSE for all noncircular ellipses?

- (A) The eccentricity, e , is less than one.
 (B) An ellipse has two foci.
 (C) The sum of the two distances from the two foci to any point on the ellipse is $2a$ (i.e., twice the semimajor distance).
 (D) The coefficients A and C preceding the x^2 and y^2 terms in the general form of the equation are equal.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

19. What is the radius of a sphere with a center at the origin and that passes through the point (8, 1, 6)?

- (A) 10
 (B) 65
 (C) $\sqrt{101}$
 (D) 100

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{8^2 + 1^2 + 6^2}$$

$$= 10.050 (\sqrt{101})$$

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= a_x b_x + a_y b_y + a_z b_z \\ &= (4)(24) + (12)(-8) + (6)(6) \\ &= 36 \end{aligned}$$

$$(14)(26)\cos\theta = 36$$

$$\cos\theta = 0.0989$$

$$\theta = \cos^{-1} 0.0989$$

$$= 84.32^\circ$$

Answer is B.

FE-STYLE EXAM PROBLEMS

Mathematics

1. What value of A satisfies the expression $A^{-6/8} = 0.001$?

- (A) 0
- (B) 100
- (C) 1000
- (D) 10,000

2. What expression is equivalent to $\log(x/(y+z))$?

- (A) $\log x - \log y - \log z$
- (B) $\log x - \log(y+z)$
- (C) $\frac{\log x}{\log y + \log z}$
- (D) $e^{x/(y+z)}$

$x = 1.2$
 $y = 3.4$
 $z = 5.6$ } substitute + check

Problem 3 through Prob. 5 refer to the following system of equations.

$$\begin{aligned} 10x + 3y + 10z &= 5 \\ 8x - 2y + 9z &= 5 \\ 8x + y - 10z &= 5 \end{aligned}$$

3. What is the cofactor matrix of the coefficient matrix?

(A) $\begin{bmatrix} 11 & 152 & 24 \\ 40 & -180 & 14 \\ 47 & -10 & -44 \end{bmatrix}$

(B) $\begin{bmatrix} 11 & -152 & 24 \\ -40 & 180 & 14 \\ 47 & 10 & 44 \end{bmatrix}$

(C) $\begin{bmatrix} 29 & -8 & -8 \\ -20 & -20 & 34 \\ 7 & 170 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 29 & 8 & -8 \\ 20 & -20 & -34 \\ 7 & -170 & 4 \end{bmatrix}$

4. What is the classical adjoint of the coefficient matrix?

(A) $\begin{bmatrix} 29 & -20 & 7 \\ -8 & -20 & 170 \\ -8 & 34 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 11 & 40 & 47 \\ 152 & -180 & -10 \\ 24 & 14 & -44 \end{bmatrix}$

(C) $\begin{bmatrix} 29 & 20 & 7 \\ 8 & -20 & -170 \\ -8 & -34 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 40 & 47 & 160 \\ 144 & -140 & 1800 \\ 32 & 658 & 18 \end{bmatrix}$

Basic Matrix Knowledge

$$A = \begin{pmatrix} 1 & 3 & 2 \\ -4 & 6 & 5 \\ 0 & 2 & 1 \end{pmatrix} \quad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \leftarrow \text{Sign matrix}$$

$$\begin{pmatrix} -4 & +4 & -8 \\ +1 & 1 & -2 \\ 3 & -13 & 18 \end{pmatrix} \leftarrow \text{co-factor matrix}$$

$$\begin{pmatrix} -4 & 1 & 3 \\ 4 & 1 & -13 \\ -8 & -2 & 18 \end{pmatrix} \leftarrow \text{(classical) adjoint}$$

$$\det \begin{pmatrix} + & - & + \\ 1 & 3 & 2 \\ -4 & 6 & 5 \\ 0 & 2 & 1 \end{pmatrix} = +1 \cdot (-4) - 3(-4) + 2(-8)$$

$$= -8 \leftarrow \text{determinant}$$

$$\frac{\begin{pmatrix} -4 & 1 & 3 \\ 4 & 1 & -13 \\ -8 & -2 & 18 \end{pmatrix}}{-8} = \begin{pmatrix} -\frac{1}{2} & \frac{1}{8} & \frac{3}{8} \\ \frac{1}{2} & \frac{1}{8} & -\frac{13}{8} \\ -1 & -\frac{1}{4} & \frac{9}{4} \end{pmatrix} \leftarrow \text{Inverse, } A^{-1}$$

5. What is the inverse of the coefficient matrix?

(A) $\begin{bmatrix} 0.014 & -0.050 & 0.058 \\ -0.189 & 0.223 & 0.012 \\ 0.030 & 0.017 & 0.055 \end{bmatrix}$

(B) $\begin{bmatrix} 0.032 & 0.022 & 0.008 \\ 0.009 & -0.022 & -0.188 \\ -0.009 & -0.038 & 0.004 \end{bmatrix}$

(C) $\begin{bmatrix} \frac{29}{906} & \frac{-10}{453} & \frac{7}{906} \\ \frac{-4}{453} & \frac{-10}{453} & \frac{85}{453} \\ \frac{-4}{453} & \frac{17}{453} & \frac{2}{453} \end{bmatrix}$

(D) $\begin{bmatrix} \frac{11}{806} & \frac{20}{403} & \frac{47}{806} \\ \frac{76}{403} & \frac{-90}{403} & \frac{-5}{403} \\ \frac{12}{403} & \frac{7}{403} & \frac{-22}{403} \end{bmatrix}$

For the following problems, use the NCEES Handbook as your only reference.

6. What is the solution to the following system of simultaneous linear equations?

$$10x + 3y + 10z = 5$$

$$8x - 2y + 9z = 3$$

$$8x + y - 10z = 7$$

(A) $x=0.326; y=-0.192; z=0.586$

(B) $x=0.148; y=1.203; z=0.099$

(C) $x=0.625; y=0.186; z=-0.181$

(D) $x=0.282; y=-1.337; z=-0.131$

7. Find the length of the resultant of the following vectors.

$$3\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$$

$$7\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

$$-16\mathbf{i} - 14\mathbf{j} + 2\mathbf{k}$$

(A) 3

(B) 4

(C) 10

(D) 14

8. Find the unit vector (i.e., the direction vector) associated with the vector $18\mathbf{i} + 3\mathbf{j} + 29\mathbf{k}$.

(A) $0.525\mathbf{i} + 0.088\mathbf{j} + 0.846\mathbf{k}$

(B) $0.892\mathbf{i} + 0.178\mathbf{j} + 0.416\mathbf{k}$

(C) $1.342\mathbf{i} + 0.868\mathbf{j} + 2.437\mathbf{k}$

(D) $6\mathbf{i} + \mathbf{j} + \frac{29}{3}\mathbf{k}$

$\frac{\vec{v}}{|\vec{v}|} = \text{unit vector}$

9. What is the cross product, $\mathbf{A} \times \mathbf{B}$, of vectors \mathbf{A} and \mathbf{B} ?

$$\mathbf{A} = \mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$

$$\mathbf{B} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$$

(A) $\mathbf{i} - \mathbf{j} - \mathbf{k}$

(B) $-\mathbf{i} + \mathbf{j} + \mathbf{k}$

(C) $2\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$

(D) $2\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$

10. What is the value of $\ln(7.3891^{xy})$?

(A) $2/xy$

(B) $0.5xy$

(C) $0.8686xy$

(D) $2xy$

11. If the \log_{10} of 4 is $0.703x$, what is the \log_{10} of $1/4$?

(A) $-1.703x$

(B) $-0.703x$

(C) $0.297x$

(D) $0.703x$

$\log 4 = -\log \frac{1}{4}$

12. What are the polar (r, θ) coordinates of the point that has rectangular (x, y) coordinates of $(4, 6)$?

(A) $(4, 6^\circ)$

(B) $(4, 56.3^\circ)$

(C) $(7.21, 33.7^\circ)$

(D) $(7.21, 56.3^\circ)$

$x = r \cos \theta$

$y = r \sin \theta$

$r^2 = \sqrt{x^2 + y^2}$

$\theta = \arctan(y/x)$

13. The polar (r, θ) coordinates of a point are $(4, 120^\circ)$. What are the rectangular (x, y) coordinates?

(A) $(2, 3.46)$

(B) $(3.46, 2)$

(C) $(-2, 3.46)$

(D) $(-2, -3.46)$

14. What is the matrix product \mathbf{AB} of matrices \mathbf{A} and \mathbf{B} ?

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

- (A) $\begin{bmatrix} 10 & 4 \\ 7 & 3 \end{bmatrix}$
 (B) $\begin{bmatrix} 11 & 4 \\ 5 & 2 \end{bmatrix}$
 (C) $\begin{bmatrix} 8 & 3 \\ 2 & 0 \end{bmatrix}$
 (D) $\begin{bmatrix} 10 & 7 \\ 4 & 3 \end{bmatrix}$

15. What is the matrix product \mathbf{AB} of matrices \mathbf{A} and \mathbf{B} ?

$$\mathbf{A} = [1 \ 2 \ 3] \quad \mathbf{B} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$$

- (A) $\begin{bmatrix} 2 \\ -6 \\ 12 \end{bmatrix}$
 (B) [8]
 (C) [20]
 (D) [2 -6 12]

16. What is the matrix difference $\mathbf{A} - \mathbf{B}$ of matrices \mathbf{A} and \mathbf{B} ?

$$\mathbf{A} = \begin{bmatrix} 3 & 4 & 1 \\ 5 & 2 & -2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 5 & 2 & -6 \\ -5 & 4 & 2 \end{bmatrix}$$

- (A) $\begin{bmatrix} -2 & 2 & 5 \\ 0 & -2 & -4 \end{bmatrix}$
 (B) $\begin{bmatrix} 2 & -2 & -7 \\ -10 & 2 & 4 \end{bmatrix}$
 (C) $\begin{bmatrix} 2 & 2 & -5 \\ 10 & -2 & -4 \end{bmatrix}$
 (D) $\begin{bmatrix} -2 & 2 & 7 \\ 10 & -2 & -4 \end{bmatrix}$

17. The determinant of matrix \mathbf{A} is -5 . What is the missing value a ?

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & 2 \\ 3 & -1 & a \end{bmatrix}$$

- (A) -9
 (B) $5/22$
 (C) 7
 (D) 17

*substitute each in a
compute det on TI to
see which gives you -5 .*

18. If the determinant of matrix \mathbf{A} is -40 , what is the determinant of matrix \mathbf{B} ?

$$\mathbf{A} = \begin{bmatrix} 4 & 3 & 2 & 1 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & -1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 2 & 1.5 & 1 & 0.5 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & -1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

- (A) -80
 (B) -40
 (C) -20
 (D) 0.5

*$B = \frac{1}{2}A$, so
 $\det(B) = \frac{1}{2} \det(A)$*

19. What is the determinant of matrix \mathbf{A} ?

$$\mathbf{A} = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

- (A) 0
 (B) 15
 (C) 14
 (D) 26

20. What is the inverse of matrix A ?

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$$

- (A) $\begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$
- (B) $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$
- (C) $\begin{bmatrix} 1 & -3 \\ -1 & 2 \end{bmatrix}$
- (D) $\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

21. What is the cube root of the complex number $(8, 60^\circ)$?

- (A) $(2)(\cos 60^\circ + i\sin 60^\circ)$
- (B) $(2)(i\cos 20^\circ + \sin 20^\circ)$
- (C) $(2.7)(\cos 20^\circ + i\sin 20^\circ)$
- (D) $(2)(\cos(20^\circ + 120^\circ n) + i\sin(20^\circ + 120^\circ n))$

22. Which of the following statements IS FALSE for the identity matrix, I ?

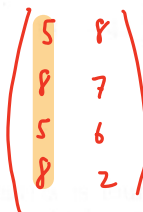
- (A) I is a square matrix.
- (B) All entries in I are zero, except the entries whose row and column numbers are equal.
- (C) All nonzero entries are equal to 1.
- (D) $AI = A^{-1}$

23. What is the transpose of matrix A ?

$$A = \begin{bmatrix} 5 & 8 & 5 & 8 \\ 8 & 7 & 6 & 2 \end{bmatrix}$$

Make rows into columns

- (A) $\begin{bmatrix} 8 & 7 & 6 & 2 \\ 5 & 8 & 5 & 8 \end{bmatrix}$
- (B) $\begin{bmatrix} 2 & 6 & 7 & 8 \\ 8 & 5 & 8 & 5 \end{bmatrix}$
- (C) $\begin{bmatrix} 8 & 5 \\ 7 & 8 \\ 6 & 5 \\ 2 & 8 \end{bmatrix}$
- (D) $\begin{bmatrix} 5 & 8 \\ 8 & 7 \\ 5 & 6 \\ 8 & 2 \end{bmatrix}$



24. What is the sum of the following finite sequence of terms?

$$18, 25, 32, 39, \dots, 67$$

- (A) 181
- (B) 213
- (C) 234
- (D) 340

25. What is the sum of the following finite sequence of terms?

$$32, 80, 200, \dots, 19531.25$$

- (A) 21,131.25
- (B) 24,718.25
- (C) 31,250
- (D) 32,530.75

26. Which of the following statements is true for a power series with the general term $a_i x^i$?

- I. An infinite power series converges for $x < 1$.
- II. Power series can be added together or subtracted within their interval of convergence.
- III. Power series can be integrated within their interval of convergence.

- (A) I only
- (B) II only
- (C) I and III
- (D) II and III

27. Expand the function $f(x)$ about $a=0$ to obtain $f(b)$. What are the first two terms of the Taylor series?

$$f(x) = \frac{1}{3x^3 + 4x + 8}$$

(A) $\frac{1}{16} + \frac{b}{8}$

(B) $\frac{1}{8} - \frac{b}{16}$

(C) $\frac{1}{8} + \frac{b}{16}$

(D) $\frac{1}{4} - \frac{b}{16}$

$f(0) \cdot b^0 + f'(0) \cdot b^1$
 $\frac{1}{8} b^0 + -0.0625 \cdot b^1$

Found on TI-36X Pro
 $= \frac{1}{16}$

28. Which of the following choices is closest to the rationalized form of the complex number?

$$\frac{7 + j5.2}{3 + j4}$$

(A) $-0.030 + j1.8$

(B) $1.67 - j0.5$

(C) $2.33 + j1.2$

(D) $2.33 + j1.3$

$= 1.672 - 0.496j$
 on TI-36X

Problem 29 and Prob. 30 refer to the vectors \mathbf{A} and \mathbf{B} .

$$\mathbf{A} = 2\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}$$

$$\mathbf{B} = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$$

29. What is the dot product, $\mathbf{A} \cdot \mathbf{B}$, of the vectors?

(A) $-4\mathbf{i} + 4\mathbf{j} - 32\mathbf{k}$

(B) $-4\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$

(C) -40

(D) -32

30. What is the cross product, $\mathbf{A} \times \mathbf{B}$, of the vectors?

(A) $-24\mathbf{i} - 8\mathbf{j} + 10\mathbf{k}$

(B) $-24\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}$

(C) $-4\mathbf{i} - 4\mathbf{j} - 32\mathbf{k}$

(D) $-4\mathbf{i} + 4\mathbf{j} - 32\mathbf{k}$

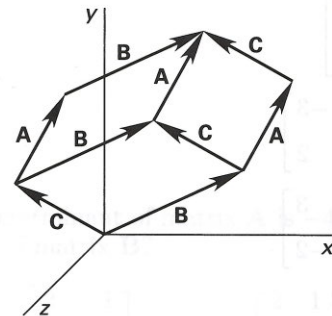
31. What is the volume of a parallelepiped with sides represented by the zero-based vectors \mathbf{A} , \mathbf{B} , and \mathbf{C} ?

$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$
 addition to
 formula sheet

$$\mathbf{A} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

$$\mathbf{B} = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{C} = \mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$$



(A) 14

(B) 28

(C) 35

(D) 42

32. The cofactor matrix of matrix \mathbf{A} is \mathbf{C} . What is the inverse of matrix \mathbf{A} ?

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 6 & -8 & -1 \\ -5 & 10 & 0 \\ -2 & 1 & 2 \end{bmatrix}$$

(A) $\begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.50 & 0 \\ 0 & 0 & 0.25 \end{bmatrix}$

(B) $\begin{bmatrix} 0.25 & 0.50 & 0.33 \\ 0.33 & 0.50 & 0.50 \\ 0.50 & 1.0 & 0.25 \end{bmatrix}$

(C) $\begin{bmatrix} 1.2 & -1.0 & -0.40 \\ -1.6 & 2.0 & 0.20 \\ -0.20 & 0 & 0.40 \end{bmatrix}$

(D) $\begin{bmatrix} 0.80 & 0.40 & -0.60 \\ 0.20 & -0.40 & 0.40 \\ -0.40 & 0.60 & 0.80 \end{bmatrix}$

FE-STYLE EXAM PROBLEMS

1. What are the minimum and maximum values, respectively, of the equation $f(x) = 5x^3 - 2x^2 + 1$ on the interval $[-2, 2]$?

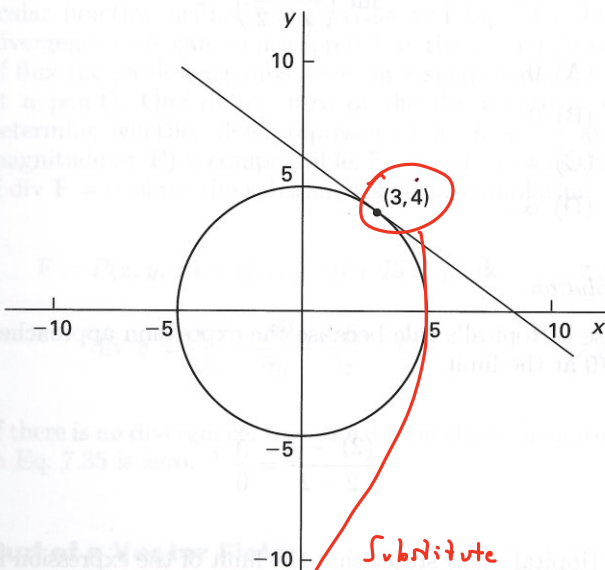
- (A) -47, 33
- (B) -4, 4
- (C) 0.95, 1
- (D) 0, 0.27

2. What is the first derivative, dy/dx , of the following expression?

- (A) 0
- (B) $\frac{-x}{y}(1 - x \ln x)$
- (C) $\frac{-y}{x}(1 + \ln xy)$
- (D) $\frac{y}{x}$

$\ln(xy)^x = e \quad x \cdot \ln(xy) = 1$
 $x \cdot \ln xy - 1 = 0$
 $\frac{dy}{dx} = \frac{-f_x}{f_y} = \frac{-(\ln xy + 1)}{\frac{x}{y}}$
 $= -\frac{y}{x}(\ln xy + 1)$

3. What is the standard form of the equation of the line tangent to a circle centered at the origin with a radius of 5 at the point (3, 4)?



- (A) $x = \frac{-4}{3}y - \frac{25}{4}$
- (B) $y = \frac{3}{4}x + \frac{25}{4}$
- (C) $y = \frac{-3}{4}x + \frac{9}{4}$
- (D) $y = \frac{-3}{4}x + \frac{25}{4}$

Substitute $x=3, y=4$ into each to see which one(s) work

4. Evaluate dy/dx for the following expression.

On TI-36X $y = e^{-x} \sin 2x \quad \frac{dy}{dx} \Big|_{x=1.7}$

- (A) $e^{-x}(2 \cos 2x - \sin 2x)$
 - (B) $-e^{-x}(2 \sin 2x + \cos 2x)$
 - (C) $e^{-x}(2 \sin 2x + \cos 2x)$
 - (D) $-e^{-x}(2 \cos 2x - \sin 2x)$
- Substitute $x=1.7$ & compare to above

5. Evaluate the following limit. (x is in radians.)

$$\lim_{x \rightarrow \pi} \left(\frac{x^2 - \pi x + \sin x}{-\sin x} \right)$$

- (A) 0
- (B) 1
- (C) $\pi - 1$
- (D) $2(\pi - 1)$

For the following problems, use the NCEES Handbook as your only reference.

6. What is the partial derivative $\partial v / \partial y$ of the following function?

$$v = 3x^2 + 9xy - \frac{y}{\ln z} + \cos(z^2 + x)$$

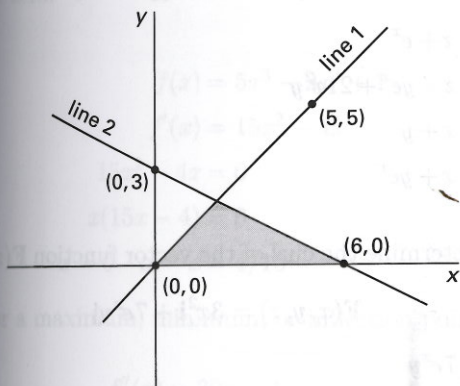
- (A) $9x - \frac{1}{\ln z}$
- (B) $6x + 9x - \frac{1}{\ln z} - \sin(z^2 + x)$
- (C) $3x^2y + \frac{9xy^2}{2} - \frac{y^2}{2 \ln z} + \frac{\sin(z^2 + x)}{z^2 + x}$
- (D) $9x + \frac{1}{\ln z}$

7. Determine the following indefinite integral.

$\int_{1.7}^{3.4} \frac{x^3 + x + 4}{x^2} dx$ on TI-36X

- (A) $\frac{x}{4} + \ln|x| - \frac{4}{x} + C \leftarrow f(3.4) - f(1.7) = ?$
- (B) $\frac{-x}{2} + \log x - 8x + C$
- (C) $\frac{x^2}{2} + \ln|x| - \frac{2}{x^2} + C$
- (D) $\frac{x^2}{2} + \ln|x| - \frac{4}{x} + C$

8. Find the shaded area between line 1, line 2, and the x-axis.



- (A) 5
- (B) 5.5
- (C) 6
- (D) 6.5

9. Evaluate the following integral.

$$\int \frac{4}{8+2x^2} dx$$

- (A) $4 \ln|8+2x^2| + C$
- (B) $\frac{1}{2} - 6x^{-3} + C$
- (C) $\arctan \frac{x}{2} + C$
- (D) $(-4)(8+2x^2)^{-2} + C$

10. What is the approximate area under the curve $y=1/x$ between $y=2$ and $y=10$?

- (A) 0.48
- (B) 1.6
- (C) 2.1
- (D) 3.0

$\int_2^{10} \frac{1}{x} dx$ on TI-36X

11. What is the approximate area bounded by the curves $y=8-x^2$ and $y=-2+x^2$?

- (A) 22.4
- (B) 26.8
- (C) 29.8
- (D) 44.7

2.236
 $\int_{-2.236}^{2.236} ((8-x^2) - (-2+x^2)) dx$
 on TI-36X ≈ 29.814

Solve $8-x^2 = -2+x^2$
 bounds of integration: $\pm \sqrt{5} \approx 2.236$

12. What is the approximate total area bounded by $y=\sin x$ over the interval $0 \leq x \leq 2\pi$? (x is in radians.)

- (A) 0
- (B) $\pi/2$
- (C) 2
- (D) 4

$\int_0^{2\pi} |\sin x| dx = 4$ on TI-36X
 (absolute value under math \rightarrow num menu)

13. What is the approximate radius of curvature of the function $f(x)$ at the point $(x, y) = (8, 16)$?

$$f(x) = x^2 + 6x - 96$$

- (A) 1.90×10^{-4}
- (B) 9.80
- (C) 96.0
- (D) 5340

14. A peach grower estimates that if he picks his crop now, he will obtain 1000 lugs of peaches, which he can sell at \$1.00 per lug. However, he estimates that his crop will increase by an additional 60 lugs of peaches for each week that he delays picking, but the price will drop at a rate of \$0.025 per lug per week. In addition, he will experience a spoilage rate of approximately 10 lugs for each week he delays. In order to maximize his revenue, how many weeks should he wait before picking the peaches?

- (A) 2 weeks
- (B) 5 weeks
- (C) 7 weeks
- (D) 10 weeks

15. What is the derivative, dy/dt , of $y = \sin^2 \omega t$?

- (A) $2\omega \sin \omega t$
- (B) $\cos^2 \omega t$
- (C) $2 \sin \omega t \cos \omega t$
- (D) $2\omega \sin \omega t \cos \omega t$

let $\omega = 1.7$
 Find $\frac{dy}{dt}$ on TI-36X
 $t = 2.4$

16. What is the slope of the curve $y=10x^2-3x-1$ when it crosses the positive part of the x-axis?

- (A) 3/20
- (B) 1/5
- (C) 1/3
- (D) 7

$10x^2 - 3x - 1 = 0$
 $x = -\frac{1}{5}, \frac{1}{2}$ (poly-solve on TI-36X)
 $\frac{dy}{dx} = 7$ (on TI-36X)
 $x = \frac{1}{2}$

17. What is the volume of revolution from $x=0$ to $x=3/2$ when the function $f(x)=2x^2$ is revolved around the y -axis?

- (A) $3\pi/2$
 (B) $27\pi/16$
 (C) $35\pi/16$
 (D) $81\pi/16$

18. What is the derivative, dy/dx , of the expression $x^2y - e^{2x} = \sin y$?

- (A) $\frac{2e^{2x}}{x^2 - \cos y}$
 (B) $\frac{2e^{2x} - 2xy}{x^2 - \cos y}$
 (C) $2e^{2x} - 2xy$
 (D) $x^2 - \cos y$

$$f(x,y) = x^2y - e^{2x} - \sin y$$

$$\frac{dy}{dx} = \frac{-f_x}{f_y} =$$

$$= -\frac{2xy - 2e^{2x} - 0}{x^2 - \cos y}$$

$$= \frac{2e^{2x} - 2xy}{x^2 - \cos y}$$

Problem 19 through Prob. 21 are based on the following statement.

A two-dimensional function, $f(x, y)$, is defined as

$$f(x, y) = 2x^2 - y^2 + 3x - y$$

19. What is the gradient vector for this function?

- (A) $\nabla f(x, y) = (2x^2 + 3x)\mathbf{i} + (-y^2 - y)\mathbf{j}$
 (B) $\nabla f(x, y) = (x^3 + \frac{3}{2}x^2)\mathbf{i} + (-\frac{1}{3}y^3 - \frac{1}{2}y^2)\mathbf{j}$
 (C) $\nabla f(x, y) = (4x + 3)\mathbf{i} + (-2y - 1)\mathbf{j}$
 (D) $\nabla f(x, y) = (3x + 4)\mathbf{i} - (2y + 1)\mathbf{j}$

20. What is the direction of the line passing through the point $(1, -2)$ that has the maximum slope?

- (A) $4\mathbf{i} + 2\mathbf{j}$
 (B) $7\mathbf{i} + 3\mathbf{j}$
 (C) $7\mathbf{i} + 4\mathbf{j}$
 (D) $9\mathbf{i} - 7\mathbf{j}$

21. What is the magnitude of the slope at the point $(1, -2)$?

- (A) 2.1
 (B) 3.5
 (C) 7.6
 (D) 8.7

22. Determine the divergence of the vector function $\mathbf{F}(x, y, z)$.

$$\mathbf{F}(x, y, z) = xz\mathbf{i} + e^xy\mathbf{j} + 7x^3y\mathbf{k}$$

- (A) $z + e^x$
 (B) $z + ye^x + 21x^2y$
 (C) $x + y$
 (D) $x + ye^x$

23. Determine the curl of the vector function $\mathbf{F}(x, y, z)$.

$$\mathbf{F}(x, y, z) = 3x^2\mathbf{i} + 7e^xy\mathbf{j}$$

- (A) $7e^xy$
 (B) $7e^xy\mathbf{i}$
 (C) $7e^xy\mathbf{j}$
 (D) $7e^xy\mathbf{k}$

24. Determine the Laplacian of the scalar function $\frac{1}{3}x^3 - 9y + 5$ at the point $(3, 2, 7)$.

- (A) 0
 (B) 1
 (C) 6
 (D) 18

25. Find dy/dx for the parametric equations given.

$$x = 2t^2 - t$$

$$y = t^3 - 2t + 1$$

- (A) $3t^2$
 (B) $3t^2/2$
 (C) $4t - 1$
 (D) $(3t^2 - 2)/(4t - 1)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{3t^2 - 2}{4t - 1}$$

Solution

This is a first-order, linear differential equation. The characteristic equation is

$$r - 5 = 0$$

The root is

$$r = 5$$

The solution is of the form

$$y = Ce^{5t}$$

The initial condition is used to find C .

$$y(0) = Ce^{5(0)} = 3$$

$$C = 3$$

$$y = 3e^{5t}$$

Answer is B.

Problem 3

What is the Laplace transform of a time-dependent function of magnitude e^{-at} ?

- (A) $\frac{1}{a}$
- (B) $\frac{a}{s}$
- (C) $\frac{1}{s+a}$
- (D) $\frac{1}{s-a}$

Solution

By definition, as shown in Eq. 8.22,

$$\begin{aligned} F(s) &= \int_0^\infty f(t)e^{-st} dt \\ f(t) &= e^{-at} \\ F(s) &= \int_0^\infty e^{-at} e^{-st} dt = \int_0^\infty e^{-(s+a)t} dt \\ &= \frac{-1}{s+a} \int_0^\infty e^{-(s+a)t} (-(s+a)) dt \\ &= \frac{-e^{-(s+a)t}}{s+a} \Big|_0^\infty \\ &= 0 - \left(-\frac{1}{s+a} \right) \\ F(s) &= \frac{1}{s+a} \end{aligned}$$

This could also be determined directly from a table.

Answer is C.

FE-STYLE EXAM PROBLEMS

1. Solve the following differential equation.

$$y'' + 4y' + 4y = 0$$

$$y(0) = 1$$

$$y'(0) = 0$$

Sub into DE to see which work

(A) $y = (1 - 2x)e^{2x}$

(B) $y = (2 - x)e^{-2x}$

(C) $y = (2 + x)e^{-2x}$

(D) $y = (1 + 2x)e^{-2x}$

Can check for each on TI-36x

2. What is the correct general solution for the following differential equation?

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

- (A) $y = C_1 \sin x - C_2 \cos x$
- (B) $y = C_1 \cos x - C_2 \sin x$
- (C) $y = C_1 \cos x + C_2 \sin x$
- (D) $y = e^{-x}(C_1 \cos x + C_2 \sin x)$

Problem 3 through Prob. 5 refer to the following equation and initial conditions.

$2y'' + 10y' + 8y = e^{-2x} - 10y' - 2y''$ Not 0, so not homogeneous

$8y = e^{-2x} - 10y' - 2y''$ doesn't count as non-linear

$y(0) = 1$

$y'(0) = -\frac{3}{2}$

3. What type of differential equation is shown?

- (A) nonlinear, second-order, nonhomogeneous
- (B) linear, second-order, homogeneous
- (C) linear, second-order, nonhomogeneous
- (D) linear, third-order, nonhomogeneous

4. Which of the following statements is true for equation?

- (A) The system defined by the equation is unstable.
- (B) The complete solution to the equation will be the sum of a complementary solution and a particular solution.
- (C) The equation may be solved by successive integrations.
- (D) An integrating factor must be used to solve the equation.

5. Which of the following is a complete solution to the equation?

- (A) $y = \frac{9}{4}e^x - \ln 2x$ *Check initial conditions*
- (B) $y = \frac{9}{4}e^x - 2e^{4x}$ *first.*
- (C) $y = \frac{41}{108}e^{-x} - \frac{11}{108}e^{-4x} + \frac{1}{36}e^{-2x}$
- (D) $y = e^{-x} + \frac{1}{4}e^{-4x} - \frac{1}{4}e^{-2x}$

For the following problems, use the NCEES Handbook as your only reference.

What is the Laplace transform of $\sin t \cos t$?

- (A) $\frac{1}{s^2 + 4}$
- (B) $\frac{1}{s + 2}$
- (C) $\frac{1}{2t + 2}$
- (D) $\frac{1}{2s + 4}$

Problem 7 and Prob. 8 refer to the Laplace transform function, $\mathcal{L}(s)$.

$$\mathcal{L}(s) = \frac{20}{s(s+10)}$$

7. What is the partial fraction expansion of $\mathcal{L}(s)$?

- (A) $\frac{1}{s} - \frac{1}{s+10}$
- (B) $\frac{1}{s} - \frac{2}{s+10}$
- (C) $\frac{2}{s} - \frac{1}{s+10}$
- (D) $\frac{2}{s} - \frac{2}{s+10}$

Substitute $s = 1.7$

and see which an equal

What is the inverse Laplace transform of $\mathcal{L}(s)$?

- (A) Ate^{-10t}
- (B) $Ae^{-t} + Be^{-10t}$
- (C) $A + Be^{-10t}$
- (D) $Ae^{-5t} \sin 5t$

Problem 9 and Prob. 10 refer to the Laplace transform function, $\mathcal{L}(s)$.

$$\mathcal{L}(s) = \frac{s(s+10)}{(s+5)(s+15)}$$

9. What is the partial fraction expansion of $\mathcal{L}(s)$?

- (A) $\frac{2.5}{s+5} - \frac{7.5}{s+15}$
- (B) $\frac{15}{s+5} - \frac{5}{s+15}$
- (C) $1 - \frac{75}{s+5} - \frac{25}{s+15}$
- (D) $1 - \frac{2.5}{s+5} - \frac{7.5}{s+15}$

Substitute $s = 1.7$ into each and original and compare.

What is the inverse Laplace transform of $\mathcal{L}(s)$?

- (A) $C_1 e^{-5t} + C_2 e^{-15t}$
- (B) $C_1 e^{-3.75t} - C_2 e^{-2t}$
- (C) $C_1 e^{-10t} + C_2 e^{-5t} + C_3 e^{-15t}$
- (D) $C_1 - C_2 e^{-5t} - C_3 e^{-15t}$

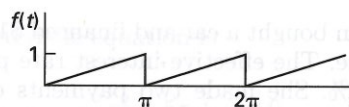
$\delta(t)$

What is the inverse transform of the Laplace transform function, $\mathcal{L}(s)$?

$$\mathcal{L}(s) = \frac{20s}{s^2 + 20s + 75}$$

- (A) $Ae^{-15t} + Be^{-5t}$
- (B) $Ate^{-7.5t}$
- (C) $Ae^{-5t} \cos 15t$
- (D) $Ae^{-15t} \cos 5t$

What are the first three sets of terms in the Fourier series of the repeating function illustrated?



- (A) $\frac{1}{2} - \cos 2t - \left(\frac{1}{2}\right) \cos 4t - \left(\frac{1}{3}\right) \cos 6t$
- (B) $\frac{1}{2} - \left(\frac{1}{\pi}\right) \sin 2t - \left(\frac{1}{2\pi}\right) \sin 4t - \left(\frac{1}{3\pi}\right) \sin 6t$
- (C) $\frac{1}{4} - \left(\frac{1}{\pi}\right) \left(\cos 2t + \sin 2t + \cos 4t + \left(\frac{1}{2}\right) \sin 4t + \cos 6t + \left(\frac{1}{3}\right) \sin 6t \right)$
- (D) $\frac{1}{4} - \left(\frac{1}{\pi}\right) \left(\left(\frac{1}{\pi}\right) \cos 2t + \sin 2t + \left(\frac{1}{2\pi}\right) \cos 4t + \left(\frac{1}{2}\right) \sin 4t + \left(\frac{1}{3\pi}\right) \cos 6t + \left(\frac{1}{3}\right) \sin 6t \right)$

13. What is the complementary solution to the following differential equation?

$$y'' - 4y' + \frac{25}{4}y = 10 \cos 8x$$

- (A) $2C_1x + C_2x - C_3x$
 - (B) $C_1e^{2x} + C_2e^{1.5x}$
 - (C) $C_1e^{2x} \cos 1.5x + C_2e^{2x} \sin 1.5x$
 - (D) $C_1e^x \tan x + C_2e^x \cot x$
- See which is a solution to $y'' - 4y' + \frac{25}{4}y = 0$ to see which works.*

14. What is the Laplace transform of the step function $f(t)$?

$$f(t) = u(t-1) + u(t-2)$$

- (A) $e^{-s} + e^{-2s}$
- (B) $\frac{e^{-s} + e^{-2s}}{s}$
- (C) $1 + \frac{e^{-2s}}{s}$
- (D) $\frac{e^s}{s} + \frac{e^{2s}}{s}$

15. A man borrows \$10,000 now at a 15% effective interest per payment period. Each loan payment totals \$3000. After the first payment is made, what is the outstanding balance on the loan?

- (A) \$7000
- (B) \$8050
- (C) \$8500
- (D) \$13,000

16. A woman bought a car and financed \$15,500 of the purchase price. The effective interest rate per payment period was 1%. She made two payments of \$350 and then decided to pay off the remaining balance in one lump sum. What was the approximate balance?

- (A) \$14,800
- (B) \$14,900
- (C) \$15,000
- (D) \$15,100

17. The values of an unknown function follow a Fibonacci number sequence. It is known that $f(1) = 4$ and $f(2) = 1.3$. What is $f(4)$?

- (A) -4.1
- (B) 0.33
- (C) 2.7
- (D) 6.6

18. What is the complete solution to the following differential equation?

$$y' + 8y = 16$$

$$y(0) = 5$$

- (A) $-11e^{-8x} + 16$
- (B) $e^{-2x} + 4$
- (C) $3e^{-8x} + 2$
- (D) $5e^{-8x} + 2$

Check initial condition first.

Then, substitute the remaining possibilities to see which one works

19. What is the inverse-transform of the z-transform function $F(z)$?

$$F(z) = \frac{z}{z-5}$$

- (A) e^{-5k}
- (B) $2k$
- (C) 2^k
- (D) 5^k

20. Find the general solution of the following differential equation.

$$(x^2 + 9) \frac{dy}{dx} = xy$$

- (A) $y = \sqrt{x^2 + 9} + C$
- (B) $y = x^2 + 9$
- (C) $y = C\sqrt{x^2 + 9}$
- (D) $y = x^2 - 9$

21. Use Euler's approximation to determine $x(1.5)$, the value of a function at 1.5, given increments of time of 0.25, $x(1) = 1$, and $dx/dt = 2x$.

- (A) 0
- (B) 0.75
- (C) 1.5
- (D) 2.25