Introduction

1. **The Issue** – infinity (no intuition)

2. **Sizes of Infinity** – there are different sizes of ∞ (Georg Cantor)

3. **Ex:** $(1 + -1) + (1 + -1) + (1 + -1) + \ldots = 0 + 0 + 0 + \ldots = 0$

   $1 + (-1 + 1) + (-1 + 1) + (-1 + \ldots = 1 + 0 + 0 + \ldots = 1$

   } Grandhi's Series

$$T = 1 + -1 + 1 + -1 + 1 + -1 + \ldots$$

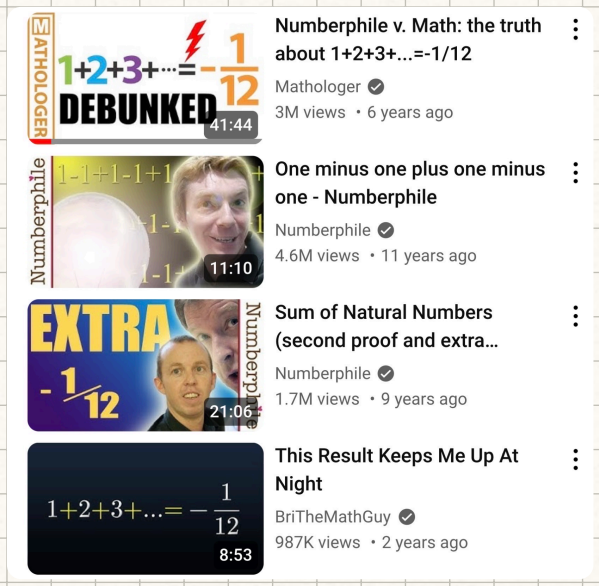
$$+ \quad T = \quad 1 + -1 + 1 + -1 + 1 + -1 + \ldots$$

$$2T = 1 + 0 + 0 + \ldots$$

$$2T = 1$$

$$\boxed{T = \tfrac{1}{2}}$$

4. **Ex:** $1 + 2 + 3 + 4 + \ldots = \cancel{\infty} = -\frac{1}{12}$



5. **Ex:** 0.999... exactly equals 1.

- If by exactly equals 1, we mean that at some point, we will have added enough 9's to get to 1, then NO.
- If by exactly equals 1, we mean there is no number between 0.999... and 1, then YES.

Number vs. Numeral

- Number → Idea that refers to a
- Numeral → Symbol we use to represent a number

concrete reality

	0	1	2	3
ADLaM	𞥐	𞥑	𞥒	𞥓
Aegean		𐄇	𐄈	𐄉
Ahom	𑜰	𑜱	𑜲	𑜳
Arabic	٠	١	٢	٣
Balinese	᭐	᭑	᭒	᭓
Bamum		𖺀	𖺁	𖺂
Bengali	০	১	২	৩
Bhaiksuki	𑱐	𑱑	𑱒	𑱓
Brahmi		𑁒	𑁓	𑁔
Chakma	𑄶	𑄷	𑄸	𑄹
Cham	꩐	꩑	꩒	꩓
Chinese (simple)	〇	一	二	三
Chinese (complex)	零	壹	貳	參

$0.999999999\ldots = 1$

𑇡 𑇢 𑇣 𑇤 𑇥 𑇦 𑇧 𑇨 𑇩 𑇪
0 1 2 3 4 5 6 7 8 9
০ ১ ২ ৩ ৪ ৫ ৬ ৭ ৮ ৯
· ١ ٢ ٣ ٤ ٥ ٦ ٧ ٨ ٩
〇 一 二 三 四 五 六 七 八 九
零 壹 貳 參 肆 伍 陸 柒 捌 玖
— I II III IV V VI VII VIII IX

$$10x = 9.999999\ldots$$
$$1x = 0.999999\ldots$$
$$9x = 9.000000\ldots$$

$$9x = 9$$
$$x = 1$$