

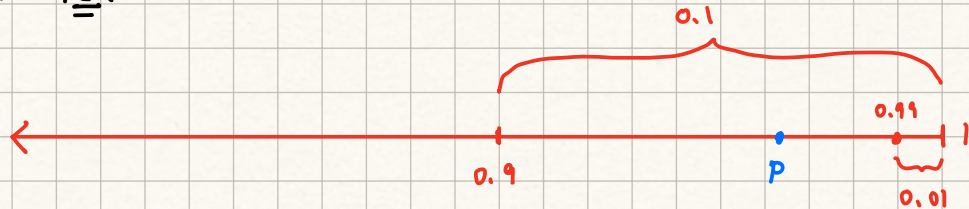
5. Ex: $0.999\dots$ exactly equals 1.

If by exactly equals 1, we mean that at some point, we will have added enough 9's to get to 1, then NO.

If by exactly equals 1, we mean there is no number between $0.999\dots$ and 1, then YES.

Number
↓
Idea that refers to a

vs. Numeral
↓
Symbol we use to represent a number



$$0.\overline{99999999} = 1$$

0	1	2	3	4	5	6	7	8	9
०	१	२	३	४	५	६	७	८	९
०	१	२	३	४	५	६	७	८	९
〇	一	二	三	四	五	六	七	八	九
零	壹	貳	參	肆	伍	陸	柒	捌	玖
I	II	III	IV	V	VI	VII	VIII	IX	

$$9x = 9$$
$$x = 1$$

7. Ex:

An Untamed Infinity

Physicists usually study quantum fields, such as the electromagnetic field, using unending sums. The first few terms in these sums get smaller and smaller, providing an approximate answer. Later terms explode, and the sum seems to lose all meaning.

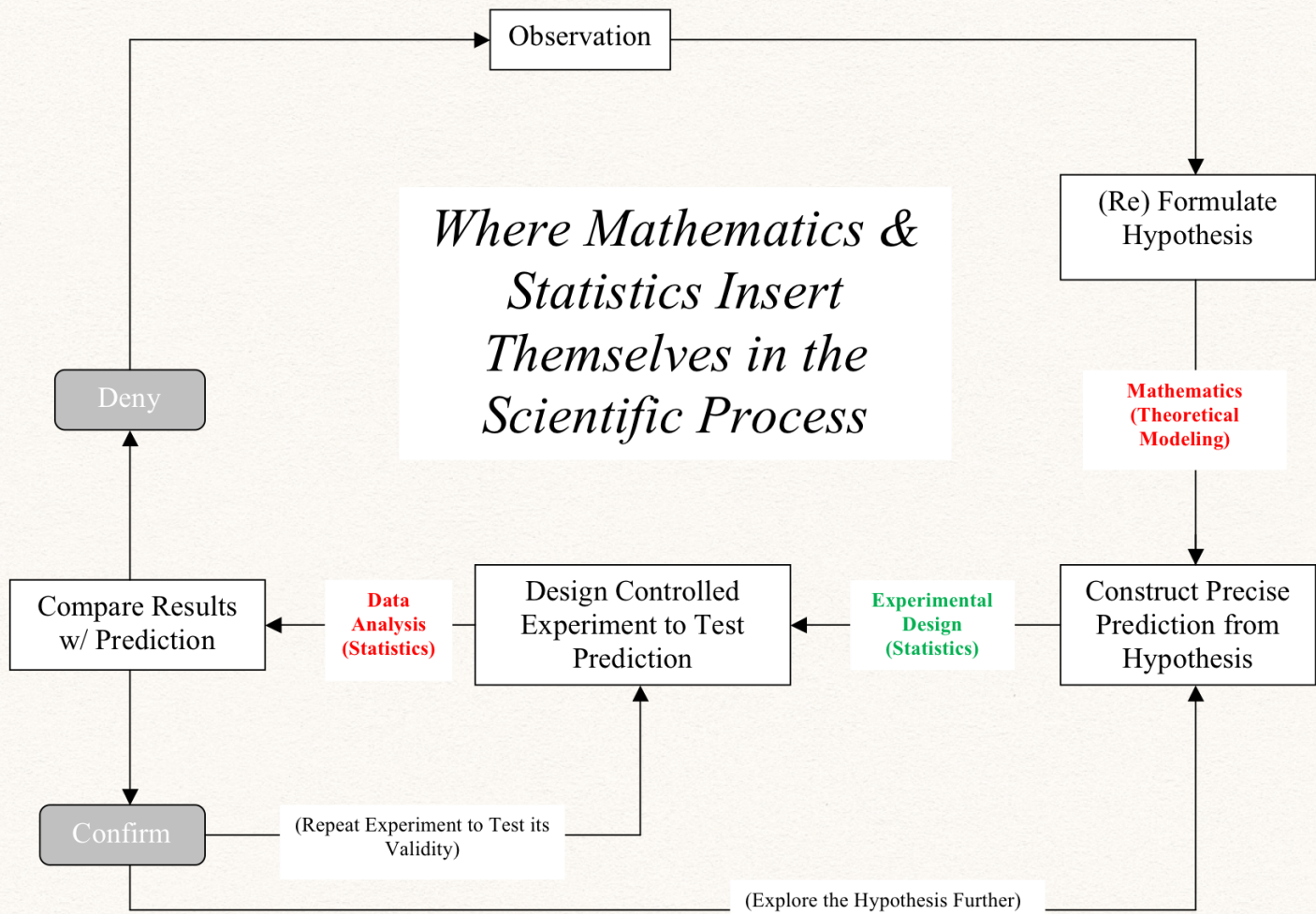
Example sum

$$a_0 + a_1x^1 + a_2x^2 + \dots \dots \dots a_{1,000}x^{1,000} + \dots$$

The diagram illustrates the divergence of a power series. On the left, the terms a_0 , a_1x^1 , and a_2x^2 are shown with arrows pointing to blocks of decreasing size, labeled "Decreasing value". On the right, the terms $a_{1,000}x^{1,000}$ and $a_n x^n$ are shown with arrows pointing to blocks of increasing size, labeled "Increasing value". A central shaded region with diagonal lines is labeled "Too hard. Most calculations stop here." and "Difficulty" with an arrow pointing right.

Understanding Modeling

1. Overview of the Scientific Method -



2. Mathematical Model - expression (equation or function) which quantitatively represents a phenomenon.

Ideally, it should be both descriptive and predictive.

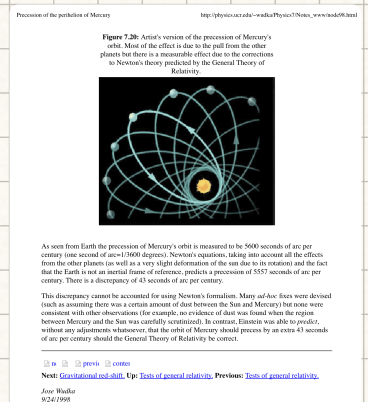
- Descriptive - accurately represents a known and measurable phenomenon.
- Predictive - accurately represents a measurable but previously unknown phenomenon.

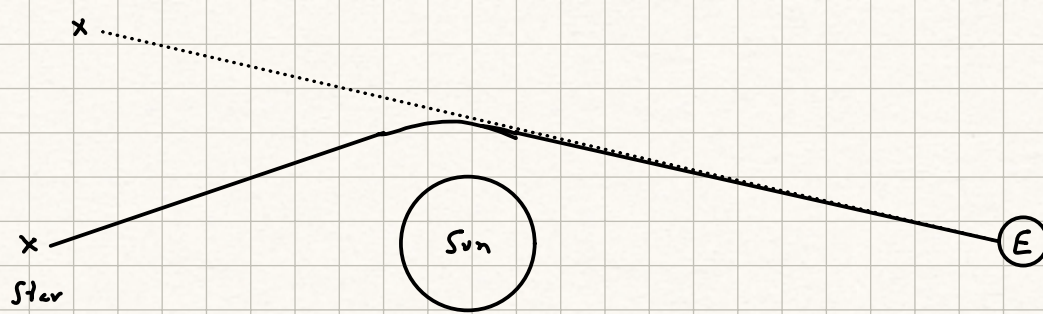
Ex: Einstein's General Theory of Relativity

- Newton - gravity is a force; derive shape of orbits of planets to be ellipses.
- Einstein - gravity is an effect of warped space.

① Descriptive - it accurately described the precession in planetary orbits.

② Predictive - anything that travels through space, including light, should be affected by gravity.





Ex: Mendeleev (1869) - Periodic Table

- Description - organised elements based on shared properties.
- Predictive - predicted the existence of previously unknown elements.

3. Statistical - uncovers useful patterns in data, and it can sometimes have predictive value, but it gives no insight into the underlying mechanisms causing a behavior.

Ex: Mendeleev + Periodic Chart - predicted existence of unknown elements but gives no insight into why elements are periodic or arranged in the pattern.

Ex: Actuarial Sciences

M/F driver's insurance rates

- No insight into why M are higher risk.
- Why?
 - Testosterone
 - Socialization
 - Frontal Lobe Development
 } underlying mechanisms

4. Theoretical Models - built from first principles (fundamental observation of physical law of universe) and they give insight into the underlying mechanisms causing a behavior.

- Proportional - two quantities are proportional iff one is a constant multiple of the other.
- Inversely Proportional - ... iff one is proportional to the reciprocal of the other.
- Jointly Proportional - ... iff one is proportional to the product of the others.