

4. Theoretical Models - built from first principles (fundamental observation of physical law of universe) and they give insight into the underlying mechanisms causing a behavior.

- Proportional - two quantities are proportional iff one is a constant multiple of the other.
- Inversely Proportional - . . . iff one is proportional to the reciprocal of the other.
- Jointly Proportional - . . . iff one is proportional to the product of the others.

Ex: P = pressure of gas in a closed container

T = temp. of the gas

$$P \propto T$$

$$P = kT$$

$k = \frac{P}{T}$ } should be constant for different values of T, P .

n = amount of gas in container (no. of moles)

$$P \propto n$$

$$P = kn$$

V = volume of the container

$$P \propto \frac{1}{V}$$

$$P = k \cdot \frac{1}{V}$$

$\therefore P$ is jointly proportional to n and T and inversely proportional to V .

$$P \propto n \cdot T \cdot \frac{1}{V}$$

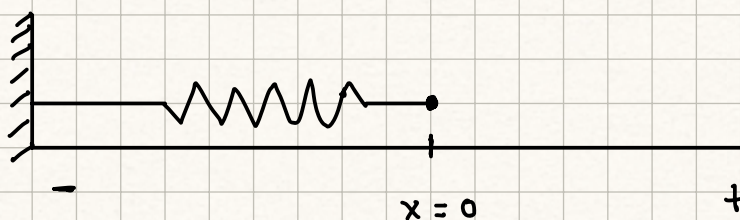
$$P = \textcolor{red}{R} \cdot n \cdot T \cdot \frac{1}{V}$$

↑
prop. constant

$$PV = nRT \quad (\text{Ideal Gas Law})$$

Ex:

Hooke's Law says that the force a spring exerts is proportional to its distortion.



$$F \propto x$$

$$F = \textcolor{red}{k} \cdot x$$

↑

proportionality constant

What factors affect the value of k ?

[List everything that might affect F]

- x {
- distortion
- k {
- type of material
 - temperature
 - tightness of the coil
 - size of the material that coiled
 - length
 - other stuff we haven't thought of...

k is constant with respect to the other variables in the equation: F, x

List of factors affecting k tell us when the model is valid.

How do we find the value of k ?

$$F = kx, \text{ so } k = \frac{F}{x}$$

x	F	$k = \frac{F}{x}$
1.2	3.8	3.17
3.6	13.4	3.72
4.8	17.9	3.73
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮

} roughly constant;
 $\bar{k} = \frac{3.17 + 3.72 + 3.73}{3}$
 $= 3.54$

Ex: Epidemiology - risk R of contracting an airborne illness is jointly proportional to the concentration C of the pathogen in the air and the time of exposure, T .

$$R \propto C \cdot T$$

$$R = k C \cdot T$$

↑
List factors affecting k

[List ... R]

CT {

- concentration
- time of exposure

K {

- age
- strength of your immune system
- air currents / filtering
- contagiousness of the pathogen
- size of room
- immunization
- natural immunity
- sleep
- diet + lifestyle

Ex: Allometry

$$SA \propto M?$$

$$SA = k \cdot M?$$

Relationship between M and V

$$\text{density, } \rho = \frac{M}{V} \rightarrow V = \frac{1}{\rho} \cdot M$$

Relationship between SA and V

$$cm^2 = (cm^3)^{\frac{2}{3}}$$

$$SA \propto V^{\frac{2}{3}}$$

$$\therefore SA \propto V^{\frac{2}{3}} \propto M^{\frac{2}{3}}$$

$$SA \propto M^{\frac{2}{3}}$$

$$SA = k \cdot M^{\frac{2}{3}}$$

↑
what affects this value?