2. Rate of Change –

Ex: Deborah Number (example of fractions being used to compare 2 quantities)

$$D = \frac{T_R}{T_O} = \frac{\text{Time of Relaxation}}{\text{Time of Observation}} = \begin{cases} \text{Large if } T_R \gg T_O & (\text{Solid}) \\ \text{Small if } T_R \ll T_O & (\text{Liquid}) \end{cases}$$

Ex:

$$C = 6.37\,q + 38$$

C is <u>not</u> proportional to q.

However, ΔC <u>is</u> proportional to Δq

$$\Delta C = 6.37 \cdot \Delta q$$

$$\frac{\Delta C}{\Delta q} = 6.37 \quad (\text{slope})$$

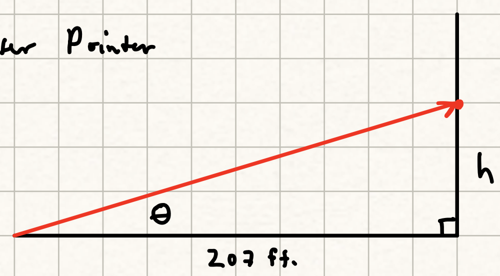
This means then any increase in g will correspond to an increase in C that is 6.37 times as large as the increase in g.

Rate of Change of C with respect to g $= \dfrac{\Delta C}{\Delta g} = 6.37$

- 6.37 is a multiplier (sensitivity).
- Δg is an error in the measurement of g.
- Δg is variability in g.

For example, if we measure $g = 2.7 \pm 0.03$ cm, then the induced error in our computed value of C would be $\Delta C = 6.37 \cdot \Delta g = 6.37(\pm 0.03) = \pm 0.191$ cm.

Ex: Laser Pointer



$$\frac{\Delta h}{\Delta \theta} = 29.2 \ \frac{\text{ft}}{\text{deg}}$$

$$\Delta h = 29.2 \cdot \Delta \theta$$

multiplier

variability in θ (hand shaking)

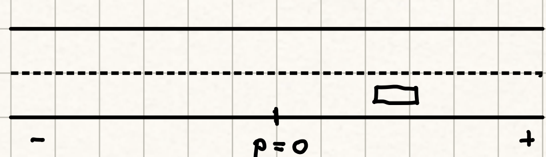
(Error enhancing situation)

Ex: Mouse Sensitivity

Note:

RoC
- $+$, large – sensitive, both quantities rise + fall together
- $+$, small – insensitive, both quantities rise + fall together
- 0 – quantities don't affect one another.
- $-$, small – insensitive, quantities rise + fall opposite one another
- $-$, large – sensitive, quantities rise + fall opposite one another

Ex: Let p = position of a car on a road, in ft., at any given time t, in seconds.



$$\frac{\Delta p}{\Delta t} = 8.7 \ \frac{ft}{s} \quad (\text{speed, velocity})$$

$$\Delta p = 8.7 \ \frac{ft}{\cancel{s}} \cdot \Delta t \ \cancel{s} = 8.7 \cdot \Delta t \ ft$$

Ex: Let p = pressure (in atm) at a depth of d meters below water.

① If $p(5) = 1.8$, give the units on the numbers and explain the meaning of the statement.

$$p(\underset{m}{5}) = \underset{atm}{1.8}$$

When depth is 5 m, the pressure is 1.8 atm.

② Do we expect the ROC to be + or − ? Explain.

Positive, since a depth inc., pressure inc. as well.

③ Suppose the ROC of $\underset{\text{output}}{p}$ wrt $\underset{\text{input}}{d}$ is 1.2. Give the units on the number & explain the meaning of the statement.

$$\frac{\Delta \text{ out}}{\Delta \text{ in}} = \frac{\Delta p}{\Delta d} = \frac{atm}{m}$$

$$\frac{\Delta p}{\Delta d} = 1.2$$

$$\Delta p = 1.2 \cdot \Delta d$$

any inc. in depth corresponds to an inc. in pressure that is 1.2 times as large as the inc. in depth.

④ If $p(3) = 1.7$ and the ROC of p wrt d is 0.6, estimate the values of $p(3.5)$ and $p(2.8)$.

$$\frac{\Delta p}{\Delta d} = 0.6 \longrightarrow \Delta p = 0.6 \cdot \Delta d$$

- $\underline{p(3.5)}$:

$$\Delta d = 3.5 - 3 = 0.5 \ m$$

$$\Delta p = 0.6 \cdot \Delta d = 0.6 \cdot 0.5 = 0.3 \ atm$$

$$p(3.5) = p(3) + 0.3 = 1.7 + 0.3 = 2.0 \ atm$$

- $p(2.8)$:

$$\Delta d = 2.8 - 3 = -0.2 \text{ m}$$

$$\Delta p = 0.6 \cdot \Delta d = (0.6)(-0.2) = -0.12 \text{ atm}$$

$$p(2.8) = p(3) + -0.12 = 1.7 - 0.12 = 1.58 \text{ atm}$$

⑤ If $p(3) = 1.7$ and ROC of p wrt d is $0.6 \frac{\text{atm}}{\text{m}}$,

- If we measure $d = 3 \pm 0.4$ m, find the induced error in our calculation of p.

$$\frac{\Delta p}{\Delta d} = 0.6 \longrightarrow \Delta p = 0.6 \cdot \Delta d$$

$$= 0.6 \cdot (\pm 0.4)$$

$$= \pm 0.24 \text{ atm}$$

- If we wish to maintain a pressure of 1.7 ± 0.2 atm on our object, how tightly must we control the depth of our object.

$$\Delta p = 0.6 \cdot \Delta d$$

$$\pm 0.2 = 0.6 \cdot \Delta d$$

$$\frac{\pm 0.2}{0.6} = \Delta d$$

$$\pm 0.33 \text{ m} = \Delta d$$