Derivatives

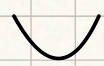
1. Recall:

- With a linear function, $y = mx + b$, m represents
  - slope [geometric]
  - rate of change [physical]

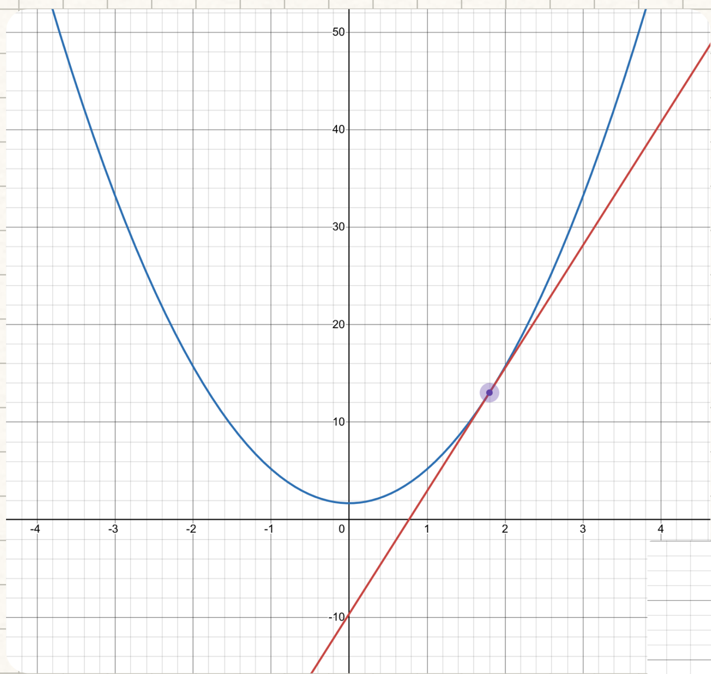
- Nonlinear functions have infinitely many different rates of change (slopes):

  Ex: $y = (3x)x$

  $= 3x^2$

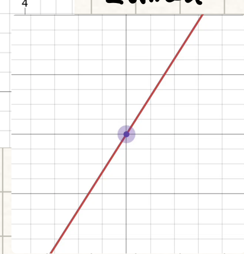


- **Question:** How do we best think of these rates of change.
  - compute
  - interpret



If we zoom in far enough on a non-linear function, it looks linear.

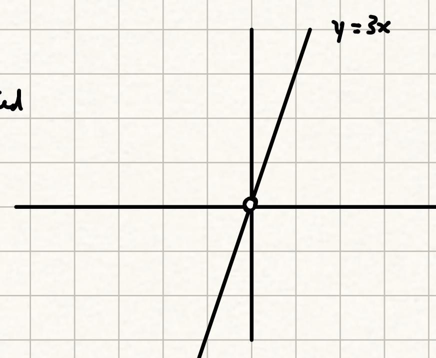
Zoomed



- **Note:** $\frac{3x^2}{x} \neq 3x$

  $f(x) = \frac{3x^2}{x}$ $\quad$ $f(1) = 3$, $f(2) = 6$, $f(0) = \frac{0}{0}$ = undefined

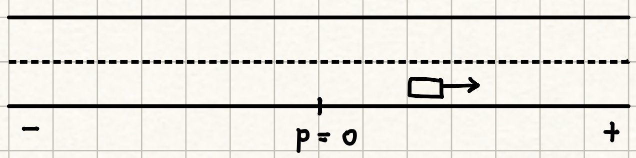
  $g(x) = 3x$ $\quad$ $g(1) = 3$, $g(2) = 6$, $g(0) = 0$



- **Note:** Power of Algebra

  $$\frac{3x + 12}{3} = \frac{3x}{3} + \frac{12}{3} = \cancel{x} + 4$$

2. Physics of Motion: Linear Motion



p = position of object (m)
t = time elapsed (s)

$$p = \underset{\uparrow}{0.8}t + \underset{\uparrow}{1.7}$$

initial position 1.7m

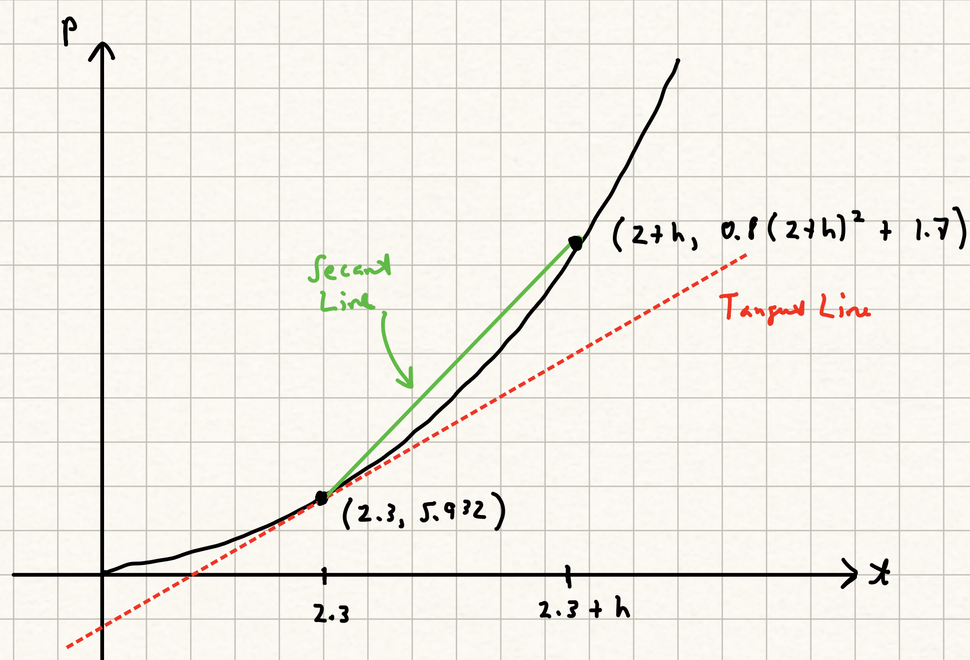
ROC = $0.8\ \frac{m}{s}$ (velocity)

Velocity is constant – always 0.8 at any time

Q: What if velocity is not constant?

$$p = (0.8t)\,t + 1.7$$

$$p = 0.8t^2 + 1.7$$



$$m = \frac{0.8(2.3+h)^2 + 1.7 - 5.932}{(2.3+h) - 2.3}$$

$$m = \frac{0.8(2.3+h)^2 + 1.7 - 5.932}{h}$$

[When $h = 0$, $\frac{0}{0}$]

$$= \frac{0.8(5.29 + 4.6h + h^2) + \overbrace{1.7 - 5.932}^{-4.232}}{h}$$

$$= \frac{\cancel{4.232} + 3.68h + 0.8h^2 - \cancel{4.232}}{h}$$

$$= \frac{h(3.68 + 0.8h)}{h}$$

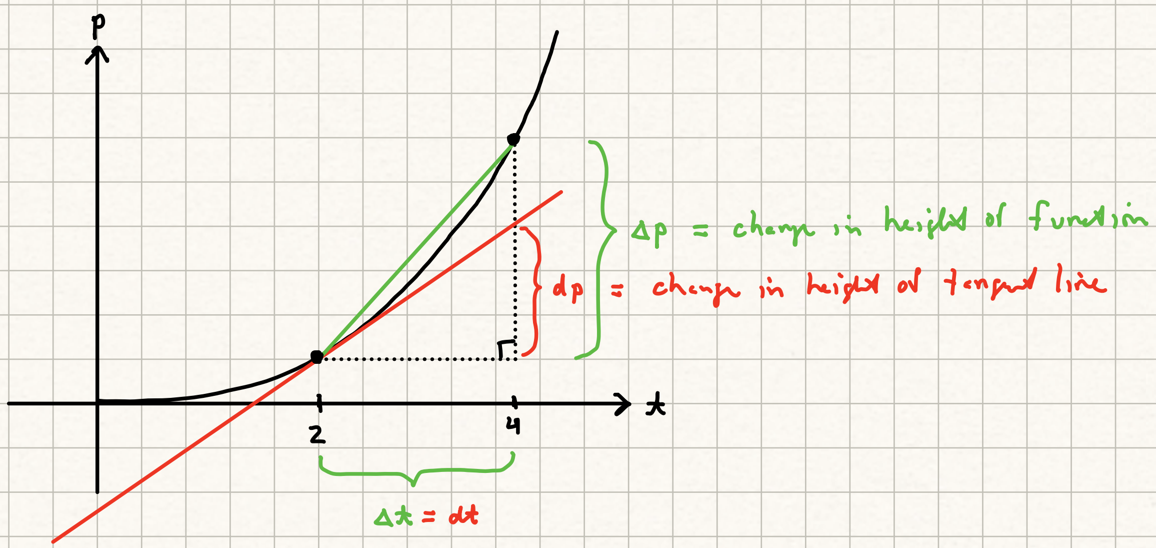
$= 3.68 + 0.8h$ (except we can let $h=0$ here).

If we could let $h=0$ in the original expression, we would get

$m = 3.68$.

Instantaneous ROC of p with respect to t at $t = 2.3s$ is $3.68\,\frac{m}{s}$.

3. Terminology -



$\frac{\Delta p}{\Delta t}$ = slope of secant line = average ROC

$\frac{dp}{dt}$ = slope of tangent line = instantaneous ROC

= derivative of p with respect to t

$= p'$

Δp and Δt are differences

dp and dt are differentials (potential infinitesimal – never 0 but always can be made as small as needed)

If we divide 2 differentials, we get a derivative.

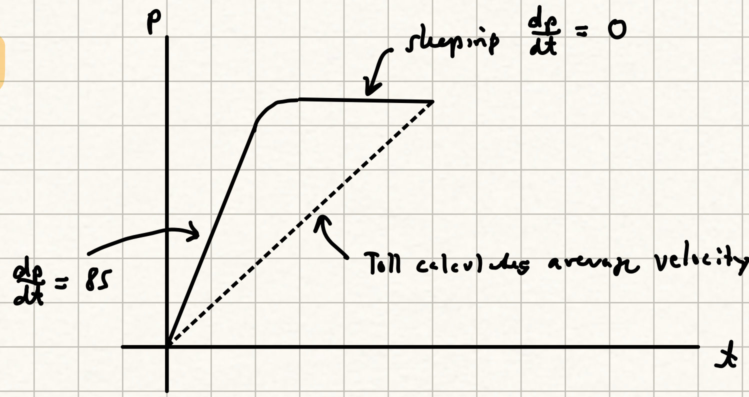
4. Ex: If $p = t^2$, find the average velocity (average ROC or slope of secant line) from $t=2$ to $t=4$.

$$m = \frac{\Delta p}{\Delta t} = \frac{p(4) - p(2)}{4 - 2} = \frac{16-4}{2} = \frac{12}{2} = 6\,\frac{m}{s}$$

Find instantaneous velocity (instantaneous ROC or slope of tangent line) at $t=2$.

$$m = \frac{dp}{dt} = \frac{p(2+h) - p(2)}{2+h-2} = \frac{(2+h)^2 - 4}{h} = \frac{\cancel{4} + 4h + h^2 - \cancel{4}}{h} = \frac{\cancel{h}(4+h)}{\cancel{h}} = 4\,\frac{m}{s}$$

Meaning:


p
sleeping $\frac{dp}{dt} = 0$
$\frac{dp}{dt} = 85$
Toll calculates average velocity
t