Practice on Interpreting Rate of Change

Mathematics 140—Applied Calculus

Dr. Peratt

Directions: Neatly complete the following problems.

1. Suppose that $r(q)$ is such that $r(25) = 3.6$ and the ROC of r with respect to q at that point is -0.2.
   (a) Estimate $r(26)$.
   (b) Estimate $r(24)$.
   (c) If $q = 25 \pm 0.5$, compute the error in the computed value of r.
   (d) Determine the tolerance on the measurement of q that would ensure that the induced error in r is no more than 0.03.
2. Suppose that $s(d)$ is such that $s(20) = 345$ and the ROC of s with respect to d at that point is 6.
   (a) Estimate $s(22)$.
   (b) Estimate $s(17)$.
   (c) If $d = 20 \pm 0.8$, compute the error induced in the computed value of s.
   (d) Determine the tolerance on the measurement of d that would ensure that the induced error in s is no more than 0.2.
3. A company's revenue from car sales, C (in thousands of dollars), is a function of the amount it spends on advertising, a (in thousands of dollars).
   (a) What does the company hope is true about the sign of the ROC of C with respect to a?
   (b) What does the statement, "The ROC of C with respect to a, when $a = 100$, is 2" mean, practically?
   (c) What does the statement, "The ROC of C with respect to a, when $a = 100$, is 0.5" mean, practically?
   (d) Suppose the company plans to spend about \$100,000 on advertising. If the ROC of C with respect to a at that point is 2, should the company spend more or less than \$100,000 on advertising? Explain. Repeat if the ROC is instead 0.5.
4. Let $h(d)$ represent the elevation, in feet, of the Mississippi River as a function of the number of miles from its source.
   (a) Which variable represents elevation?
   (b) Which variable represents distance from its source?
   (c) Interpret the meaning of the statement $h(100) = 127.5$, giving the units on each number.
   (d) Do we expect the rate of change to be positive or negative? Explain.
   (e) Do we expect the rate of change to be large or small? Explain.
   (f) If rate of change of h with respect to d is -0.3, give the units on -0.3 and interpret its meaning.
   (g) Now, suppose that, when $d = 97$, $h = 234$ and the ROC of h with respect to d is -0.3. If we change d to 110, what will h be? If we change d to 87, what will h be?
   (h) Suppose that $h(100) = 127.5$ and, at this point, the rate of change is -0.3. Find the elevation of the Mississippi 127 miles from its source and 79 miles from its source.
5. For some painkillers, the size of the dose, D (in milligrams), given depends on the weight, W (in pounds), of the patient.
   (a) Would we write $D(W)$ or $W(D)$? Explain.
   (b) Suppose $D(140) = 120$. Interpret the meaning of this statement, giving the units on each number.
   (c) Do we expect the rate of change to be positive or negative? Explain.
   (d) Suppose that, when a person weighs 150 pounds, the rate of change is 0.2. Interpret the meaning of this statement, giving the units on each number.

(e) If $D(140) = 120$ and the rate of change is 0.2, find the amount of drug that should be given to a 200 pound person.

(f) If the rate of change is 0.001, interpret the meaning of the statement, giving units for the numbers. Furthermore, rearrange your equation so that it shows how ΔD would affect ΔW. Explain what this would imply for the one prescribing the drug or the one manufacturing the drug.

6. Let $R(t)$ represent the amount of rainfall, in inches, t hours since midnight.

(a) Suppose that $R(8) = 2.3$. Interpret the meaning of this statement, giving units to the numbers.

(b) Suppose that the solution to the equation $R(t) = 1.5$ is 4. Interpret the meaning of this, giving the units on the numbers.

(c) If the storm is really beginning to pick up, so that it is beginning to rain very hard, do we expect the rate of change to be positive or negative? Large or small? Explain.

(d) If the storm is tapering off, do we expect the rate of change to be positive or negative? Large or small? Explain.

(e) When would we expect the rate of change to be zero? Negative?

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Directions: Neatly complete the following problems.

1. Suppose that $r(q)$ is such that $r(25) = 3.6$ and the ROC of r with respect to q at that point is -0.2.

   (a) Estimate $r(26)$.

   Answer: Since $\dfrac{\Delta r}{\Delta q} = -0.2$, then $\Delta r = -0.2 \cdot \Delta q = -0.2 \cdot (26 - 25) = -0.2$. Therefore, the value of $r(26) \approx 3.46 - 0.2 = 3.4$.

   (b) Estimate $r(24)$.

   Answer: Since $\dfrac{\Delta r}{\Delta q} = -0.2$, then $\Delta r = -0.2 \cdot \Delta q = -0.2 \cdot (24 - 25) = +0.2$. Therefore, the value of $r(26) \approx 3.6 + 0.2 = 3.8$.

   (c) If $q = 25 \pm 0.5$, compute the error in the computed value of r.

   Answer: The error in the measurement of q is ± 0.5, so $\Delta q = \pm 0.5$. Therefore, the error induced in $r = -0.2 \cdot \Delta q = -0.2 \cdot 0.5 = \pm 0.1$.

   (d) Determine the tolerance on the measurement of q that would ensure that the induced error in r is no more than 0.03.

   Answer: We have $\Delta r = -0.2 \cdot \Delta q$ and $\Delta r = \pm 0.03$. Therefore, $\pm 0.03 = -0.2 \cdot \Delta q$, and so $\Delta q = \pm \frac{0.03}{0.2} = \pm 0.15$.

2. Suppose that $s(d)$ is such that $s(20) = 345$ and the ROC of s with respect to d at that point is 6.

   (a) Estimate $s(22)$.

   Answer: We have $\Delta s = 6\Delta d = 6 \cdot (22 - 20) = 12$. Therefore, the value of $s(22) \approx 345 + 12 = 357$.

   (b) Estimate $s(17)$.

   Answer: We have $\Delta s = 6\Delta d = 6\dot{(}17 - 20) = -18$. Therefore, the value of $s(17) = 345 - 18 = 327$.

   (c) If $d = 20 \pm 0.8$, compute the error induced in the computed value of s.

   Answer: We have $\Delta s = 6 \cdot \Delta d = 6 \cdot (\pm 0.8) = \pm 4.8$.

   (d) Determine the tolerance on the measurement of d that would ensure that the induced error in s is no more than 0.2.

   Answer: We have $\pm 0.2 = 6 \cdot \Delta d$, so $\Delta d = \pm 0.0333\ldots$.

3. A company's revenue from car sales, C (in thousands of dollars), is a function of the amount it spends on advertising, a (in thousands of dollars).

   (a) What does the company hope is true about the sign of the ROC of C with respect to a?

   Answer: The hope it is positive, because then an increase in advertising spending, a, will result in more sales, C.

   (b) What does the statement, "The ROC of C with respect to a, when $a = 100$, is 2" mean, practically?

   Answer: Since $\Delta C = 2\Delta a$, it means that every additional dollar spent in advertising will result in a 2 dollar increase in car sales.

   (c) What does the statement, "The ROC of C with respect to a, when $a = 100$, is 0.5" mean, practically?

   Answer: Since $\Delta C = 2\Delta a$, it means that every additional dollar spent in advertising will result in only a 50 cent increase in car sales.

   (d) Suppose the company plans to spend about \$100,000 on advertising. If the ROC of C with respect to a at that point is 2, should the company spend more or less than \$100,000 on advertising? Explain. Repeat if the ROC is instead 0.5.

Answer: If the ROC is 2, then, yes, they should spend more, because they will recover the money they spent twice over. If the ROC is 0.5, then, no, they should not spend more on advertising. Only half of the money they spend will only be recovered in car sales.

4. Let $h(d)$ represent the elevation, in feet, of the Mississippi River as a function of the number of miles from its source.

   (a) Which variable represents elevation?

   Answer: h is the elevation.

   (b) Which variable represents distance from its source?

   Answer: d is the distance from the source.

   (c) Interpret the meaning of the statement $h(100) = 127.5$, giving the units on each number.

   Answer: When we are 100 miles from the source, the elevation is 127.5 feet.

   (d) Do we expect the rate of change to be positive or negative? Explain.

   Answer: We expect it to be negative, since as the distance d from the source goes up, the elevation of the river h will go down.

   (e) Do we expect the rate of change to be large or small? Explain.

   Answer: We expect it to relatively small. If it were large, we could have something more akin to rapids, but the Mississippi is fairly calm.

   (f) If rate of change of h with respect to d is -0.3, give the units on -0.3 and interpret its meaning.

   Answer: We see that $\frac{\Delta h}{\Delta d} = -0.3$, or $\Delta h = -0.3\Delta d$. This means that an increase in d will result in a decrease in h that is 0.3 times as large.

   (g) Now, suppose that, when $d = 97$, $h = 234$ and the ROC of h with respect to d is -0.3. If we change d to 110, what will h be? If we change d to 87, what will h be?

   Answer: Since $\Delta h = -0.3\Delta d$, and $\Delta d = 110 - 97 = 13$, then $\Delta h = -0.3 \cdot 13 = -3.9$. That is, h will have decreased by 3.9 feet and so it is now $234 - 3.9 = 230.1$ feet. In the second case, when $d = 87$, see that $\Delta d = 87 - 97 = -10$, so that $\Delta h = -0.3 \cdot -10 = 3.0$ feet. So, at that point, the value of h is $234 + 3 = 237$ feet.

   (h) Suppose that $h(100) = 127.5$ and, at this point, the rate of change is -0.3. Find the elevation of the Mississippi 127 miles from its source and 79 miles from its source.

   Answer: If we let $d = 127$, then $\Delta d = 127 - 100 = 27$ miles, so that $\Delta h = -0.3 \cdot 27 = -8.1$ feet. Hence, the new elevation will be $h = 127.5 - 8.1 = 119.4$ feet.

   If we let $d = 79$, then $\Delta d = 79 - 100 = -21$ miles, so that $\Delta h = -0.3 \cdot -21 = 6.3$ feet. Hence, the new elevation will be $h = 127.5 + 6.3 = 133.8$ feet.

5. For some painkillers, the size of the dose, D (in milligrams), given depends on the weight, W (in pounds), of the patient.

   (a) Would we write $D(W)$ or $W(D)$? Explain.

   Answer: We write $D(W)$, because D depends on W, so W is the input.

   (b) Suppose $D(140) = 120$. Interpret the meaning of this statement, giving the units on each number.

   Answer: This means that a person weighing 140 pounds should receive 120 mg of drug.

   (c) Do we expect the rate of change to be positive or negative? Explain.

   Answer: We expect it to be positive, because an increase in weight should correspond to an increase in drug.

   (d) Suppose that, when a person weighs 150 pounds, the rate of change is 0.2. Interpret the meaning of this statement, giving the units on each number.

   Answer: An increase in weight will result in an increase in the amount of drug administered that is 0.2 times as much as the increase in weight.

   (e) If $D(140) = 120$ and the rate of change is 0.2, find the amount of drug that should be given to a 200 pound person.

   Answer: Since $\Delta W = 60$, $\Delta D = 0.2 \cdot 60 = 12$, so that amount of drug administered should be $D = 120 + 12 = 132$ mg.

(f) If the rate of change is 0.001, interpret the meaning of the statement, giving units for the numbers. Furthermore, rearrange your equation so that it shows how ΔD would affect ΔW. Explain what this would imply for the one prescribing the drug or the one manufacturing the drug.

Answer: It means that $\Delta D = 0.001\Delta W$, so that the increase in the amount of drug administered will be only 0.001 times the amount of weight increase. Re-arranging this gives $\Delta W = 1000\Delta D$, which means that, if we increase the amount of drug a little, the amount of *additional* weight that a person would have to have in order to accommodate that change in drug would be 1000 times as much as the increase in drug. This means the process is very sensitive, so that the dosage would have to be administered extremely carefully, and the manufacturing process would have to be very precise.

6. Let $R(t)$ represent the amount of rainfall, in inches, t hours since midnight.

(a) Suppose that $R(8) = 2.3$. Interpret the meaning of this statement, giving units to the numbers.

Answer: This means that, 8 hours after midnight, 2.3 inches of rain have fallen.

(b) Suppose that the solution to the equation $R(t) = 1.5$ is 4. Interpret the meaning of this, giving the units on the numbers.

Answer: This means that $t = 4$ when $R = 1.5$. That is, at what time have 1.5 inches of rain fallen? 4 hours after midnight.

(c) If the storm is really beginning to pick up, so that it is beginning to rain very hard, do we expect the rate of change to be positive or negative? Large or small? Explain.

Answer: Since it is raining, we expect the rate of change to be positive (i.e. as time increases, the amount of rain that has fallen should increase). Since the storm is picking up, we expect that a little time passing will result in a lot more rain falling, so the rate of change should be large.

(d) If the storm is tapering off, do we expect the rate of change to be positive or negative? Large or small? Explain.

Answer: If the storm is tapering off, it is still nevertheless raining, so we expect the rate of change to be positive. However, since the storm is tapering off, the rain is light and we expect that a little time passing won't result in a lot more rain falling. The rate of change should be small.

(e) When would we expect the rate of change to be zero? Negative?

Answer: If it stops raining, then we expect no change in the amount of rain that has fallen as time increases, so the rate of change would be zero in this case. A negative rate of change would mean that, as time increases, less rain will have fallen. This will never happen, since fallen rain is a process that is not reversible. Even if the rain evaporates, it is still true that a certain amount of rain has fallen.