Practice on Finding Derivatives using Algebraic Manipulation

Applied Calculus

Dr. Peratt

Directions: Neatly complete the following problems.

1. Assume that an object in linear motion has a position $p(t) = t^2 + 9t$ at time t. Use the method shown in class to compute, exactly, the instantaneous velocity (or "instantaneous rate of change," or "derivative") of the object at time $t = 2$.

2. Assume that an object in linear motion has a position given by the function $p = 5t^2 + 7$.

   (a) Compute the average velocity (average rate of change) between time $t = 1$ and 3.

   (b) Compute the slope of the secant line between time $t = 1$ and 3.

   (c) Write the equation of the secant line between time $t = 1$ and 3.

   (d) Use the method shown in class to compute the instantaneous velocity (instantaneous rate of change) when $t = 1$.

   (e) Compute the slope of the tangent line to the graph at the point $(1, 12)$.

   (f) Write the equation of the tangent line to to graph at the point $(1, 12)$.

   (g) Use the method in class to compute the instantaneous velocity for an arbitrary point in time, t, rather than a specific numerical value of t.

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   Answer: We compute $\dfrac{\Delta p}{\Delta t} = \dfrac{p(2+h) - p(2)}{2+h-2} = \dfrac{\left[(2+h)^2 + 9(2+h)\right] - \left[2^2 + 9 \cdot 2\right]}{h} = h + 13.$ When we evaluate this last expression at $h = 0$ (so that the two points on the secant line come together), we obtain $\dfrac{dp}{dt} = 13.$

2. Assume that an object in linear motion has a position given by the function $p = 5t^2 + 7$, where p is in meters (m) and t is in seconds.

   (a) Compute the average velocity (average rate of change) between time $t = 1$ and 3.

   Answer: We compute $\dfrac{\Delta p}{\Delta t} = \dfrac{p(3) - p(1)}{3-1} = \dfrac{52 - 12}{2} = 20\dfrac{m}{s}.$

   (b) Compute the slope of the secant line between time $t = 1$ and 3.

   Answer: This is the same questions as above, so the slope of the secant line is 20.

   (c) Write the equation of the secant line between time $t = 1$ and 3.

   Answer: The slope is $m = 20$, and a point on the line is $(1, 12)$ (or, alternatively, we could use $(3, 52)$). Therefore, the equation of the line is $y - 12 = 20(x - 1)$, or, in slope-intercept form, we obtain $y = 20x - 8$.

   (d) Use the method shown in class to compute the instantaneous velocity (instantaneous rate of change) when $t = 1$.

   Answer: We compute

   $$\frac{\Delta p}{\Delta t} = \frac{p(1+h) - p(1)}{h} = \frac{(5 \cdot (1+h)^2 + 7) - (5 \cdot 1^2 + 7)}{h} = \frac{5(1 + 2h + h^2) + 7 - 12}{h} = \frac{5 + 10h + 5h^2 - 5}{h} =$$

   $$\frac{10h + 5h^2}{h} = \frac{h(10 + 5h)}{h} = 10 + 5h.$$

   When we evaluate this last expression at $h = 0$ (so that the two points on the secant line come together), we obtain an instantaneous velocity of $\frac{dp}{dt} = 10\frac{m}{s}$.

   (e) Compute the slope of the tangent line to the graph when $t = 1$.

   Answer: This is the same question as above, so the slope is 10.

   (f) Write the equation of the tangent line to to graph when $t = 1$.

   Answer: The slope is $m = 10$, and a point on the line is $(1, 12)$. (There is no other point that we can use, because this is the only point on the graph that we are given.) Therefore, the equation of the tangent line is $y - 12 = 10(x - 1)$, or, in slope-intercept form, $y = 10x + 2$.

   (g) Use the method in class to compute the instantaneous velocity for an arbitrary point in time, t, rather than a specific numerical value of t.

   Answer: We compute

   $$\frac{\Delta p}{\Delta t} = \frac{p(t+h) - p(t)}{h} = \frac{(5 \cdot (t+h)^2 + 7) - (5 \cdot t^2 + 7)}{h} = \frac{5(t^2 + 2th + h^2) + 7 - 5t^2 - 7}{h} =$$

   $$\frac{5t^2 + 10th + 5h^2 - 5t^2}{h} = \frac{10th + 5h^2}{h} = \frac{h(10t + 5h)}{h} = 10t + 5h.$$

   When we evaluate this last expression at $h = 0$ (so that the two points on the secant line come together), we obtain an instantaneous velocity of $\frac{dp}{dt} = 10t\frac{m}{s}$.