Practice on Exponential Functions

Math 140 — Applied Calculus

Dr. Peratt

Directions: Neatly complete the following problems.

1. For each exponential function, answer the questions regarding the interpretation of its parameters.
   (a) $y = 5.2(1.13)^t$.
      i. Is it growth or decay?
      ii. Explain how you know.
      iii. What is the initial value?
      iv. What is the growth/decay rate?
      v. What is the continuous growth/decay rate?
      vi. Sketch a graph of the function.
   (b) $y = -2.7(0.13)^t$.
      i. Is it growth or decay?
      ii. Explain how you know.
      iii. What is the initial value?
      iv. What is the growth/decay rate?
      v. What is the continuous growth/decay rate?
      vi. Sketch a graph of the function.
   (c) $y = -3.4(4.51)^t$
      i. Is it growth or decay?
      ii. Explain how you know.
      iii. What is the initial value?
      iv. What is the growth/decay rate?
      v. What is the continuous growth/decay rate?
      vi. Sketch a graph of the function.
   (d) $y = 3.4(4.51)^{-t}$.
      i. Is it growth or decay?
      ii. Explain how you know.
      iii. What is the initial value?
      iv. What is the growth/decay rate?
      v. What is the continuous growth/decay rate?
      vi. Sketch a graph of the function.
   (e) $y = 3.4e^{0.51t}$
      i. Is it growth or decay?
      ii. Explain how you know.
      iii. What is the initial value?
      iv. What is the growth/decay rate?
      v. What is the continuous growth/decay rate?
      vi. Sketch a graph of the function.
   (f) $y = -3.4e^{-1.27t}$
      i. Is it growth or decay?
      ii. Explain how you know.
      iii. What is the initial value?
      iv. What is the growth/decay rate?
      v. What is the continuous growth/decay rate?
      vi. Sketch a graph of the function.

2. The following functions give the populations of four towns with time t in years.

   i $P = 600(1.12)^t$

   ii $P = 1,000(1.03)^t$

   iii $P = 200(1.08)^t$

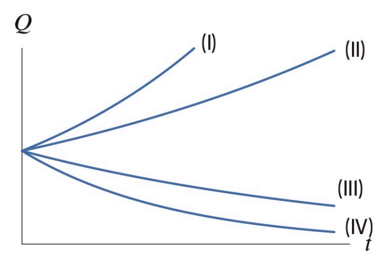
   iv $P = 900(0.90)^t$

   (a) Which town has the largest percent growth? What is the percent growth rate?

   (b) Which town ha the largest initial population? What is that initial population?

   (c) Are there any towns decreasing in size? If so, which one(s) and by what percent are they decreasing?

3. The graph in the figure below shows $Q = 50(1.2)^t$, $Q = 50(0.6)^t$, $Q = 50(0.8)^t$, and $Q = 50(1.4)^t$. Match each formula to a graph.



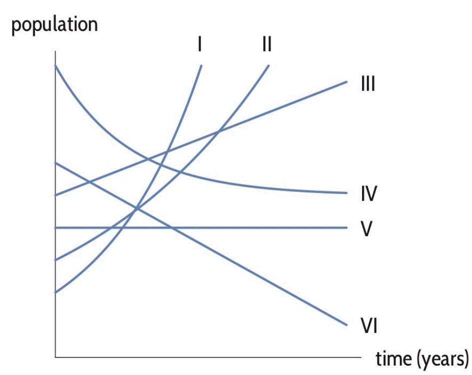
4. The figure below shows graphs of several cities' population against time. Match each of the following descriptions to a graph and write a description to match each of the remaining graphs.

   (a) The population increased by 5% per year.

   (b) The population increased by 8% per year.

   (c) The population increased by 5,000 people per year.

   (d) The population was stable.



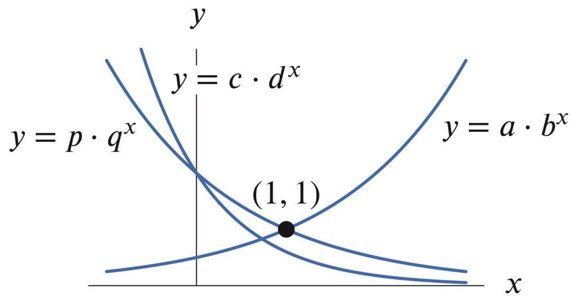
5. The exponential functions graphed below have $b,\ d,\ q$ positive.

   (a) Which of the constants $a,\ c,$ and p must be positive? Explain.

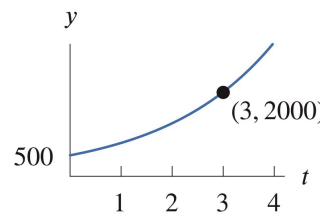
   (b) Which of the constants $a,\ b,\ c,\ d,\ p,$ and q must be between 0 and 1? Explain.

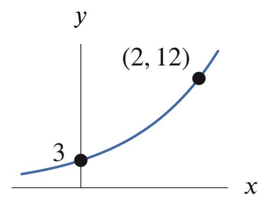
   (c) Which two of the constants $a,\ b,\ c,\ d,\ p,$ and q must be equal?

   (d) What information about the constants a and b does the point $(1, 1)$ provide?

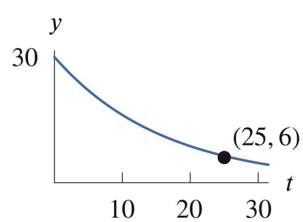


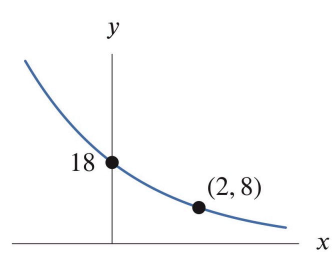
6. Give a possible formula for the functions in the figure below:





7. Give a possible formula for the functions in the figure below:





8. Write a possible formula that describes each of the following situations.

   (a) An air-freshener starts with 30 grams of material that evaporates at 2 grams per day.

   (b) An air-freshener starts with 30 grams of material that evaporates by 2% per day.

   (c) A town has a population of 1,000 people at time $t = 0$, and the population increases by 50 people every year.

(d) A town has a population of 1,000 people at time $t = 0$, and the population increases by 5% people every year.

(e) The gross domestic product, G, of Switzerland was 685.4 billion dollars in 2013 and increases by 2% per year.

(f) The gross domestic product, G, of Switzerland was 685.4 billion dollars in 2013 and increases by 7 billion dollars per year.

9. For each of the following exponential functions, sketch a quick graph to determine whether they intersect for some positive time value and, if they do, find the coordinates of that intersection.

   (a) $s = 4.7(0.57)^t$ and $s = 2.3(1.15)^t$.

   (b) $s = 2.3(0.57)^t$ and $s = 4.7(1.15)^t$.

10. A town has a population of 1,000 people at time $t = 0$, and the population increases by 5% people every year. Find the amount of time it will take for the population to double.

Practice on Exponential Functions—Solutions

Math 140 — Applied Calculus

Dr. Peratt

Directions: Neatly complete the following problems.

1. For each exponential function, answer the questions regarding the interpretation of its parameters.

   (a) $y = 5.2(1.13)^t$.

   i. Is it growth or decay?

      Answer: Growth.

   ii. Explain how you know.

      Answer: $1.13 > 1$.

   iii. What is the initial value?

      Answer: 5.2.

   iv. What is the growth/decay rate?

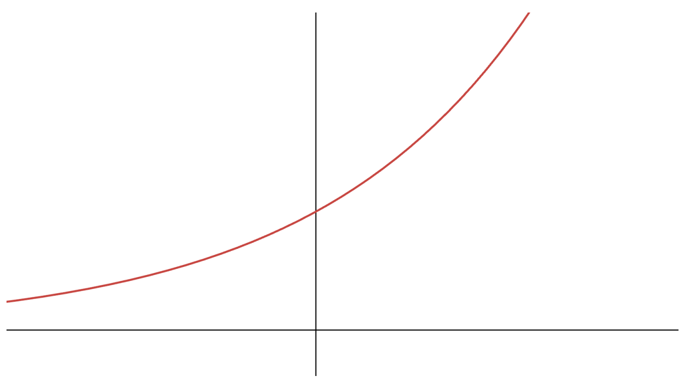
      Answer: 13%.

   v. What is the continuous growth/decay rate?

      Answer: Since $\ln(1.13) = 0.122$, the decay rate is 12.2%

   vi. Sketch a graph of the function.

      Answer: With a y-intercept of 5.2.



   (b) $y = -2.7(0.13)^t$.

   i. Is it growth or decay?

      Answer: Decay.

   ii. Explain how you know.

      Answer: $0 < 0.13 < 1$.

   iii. What is the initial value?

      Answer: -2.7.

   iv. What is the growth/decay rate?

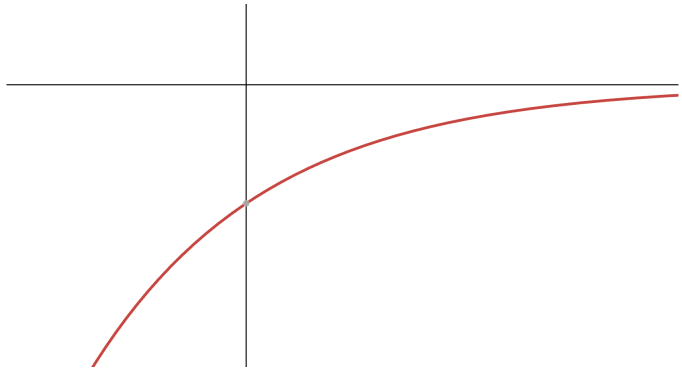
      Answer: 87%.

   v. What is the continuous growth/decay rate?

      Answer: Since $\ln(0.13) = -2.040$, the decay rate is 204%,

   vi. Sketch a graph of the function.

      Answer: With a y-intercept of -2.7.



(c) $y = -3.4(4.51)^t$

i. Is it growth or decay?

Answer: Growth.

ii. Explain how you know.

Answer: $4.51 > 1$.

iii. What is the initial value?

Answer: -3.4.

iv. What is the growth/decay rate?

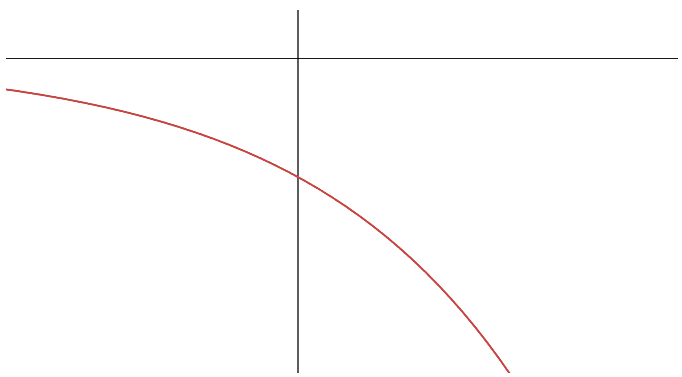
Answer: 351%.

v. What is the continuous growth/decay rate?

Answer: Since $\ln(4.51) = 1.506$, the decay rate is 150.6%.

vi. Sketch a graph of the function.

Answer: With a y-intercept of -3.4.



(d) $y = 3.4(4.51)^{-t}$.

i. Is it growth or decay?

Answer: Since $4.51 > 1$, that would indicate growth, but since the exponent is negative, that reverses things. It is Decay.

ii. Explain how you know.

Answer: $y = 3.4(4.51)^{-t} = 3.4(0.22)^t$, and $0.22 < 1$.

iii. What is the initial value?

Answer: 3.4.

iv. What is the growth/decay rate?

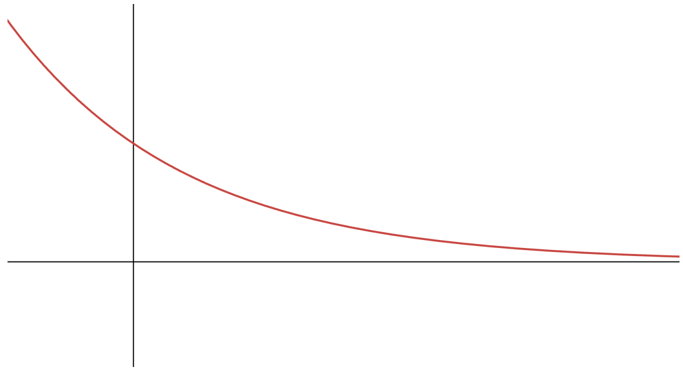
Answer: 78%.

v. What is the continuous growth/decay rate?

Answer: Since $\ln(0.22) = -1.514$, the decay rate is 151.4%.

vi. Sketch a graph of the function.

Answer: With a y-intercept of 3.4.



(e) $y = 3.4e^{0.51t}$

i. Is it growth or decay?

Answer: Growth.

ii. Explain how you know.

Answer: 0.51 is positive.

iii. What is the initial value?

Answer: 3.4.

iv. What is the growth/decay rate?

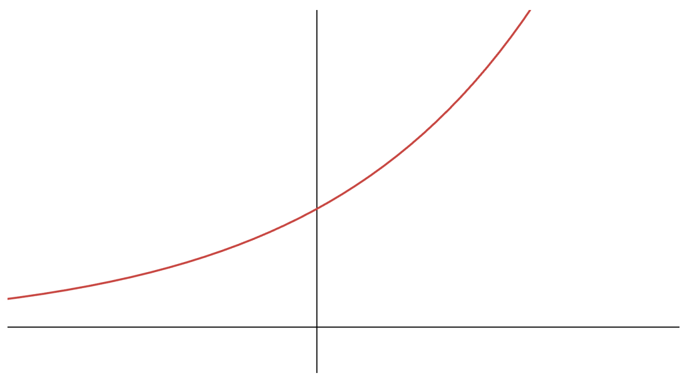
Answer: Since $3.4e^{0.51t} = 3.4(e^{0.51})^t = 3.4(1.665)^t$, the growth rate is 66.5%.

v. What is the continuous growth/decay rate?

Answer: 51%.

vi. Sketch a graph of the function.

Answer: With a y-intercept of 3.4.



(f) $y = -3.4e^{-1.27t}$

i. Is it growth or decay?

Answer: Decay.

ii. Explain how you know.

Answer: -1.27 is negative.

iii. What is the initial value?

Answer: -3.4.

iv. What is the growth/decay rate?

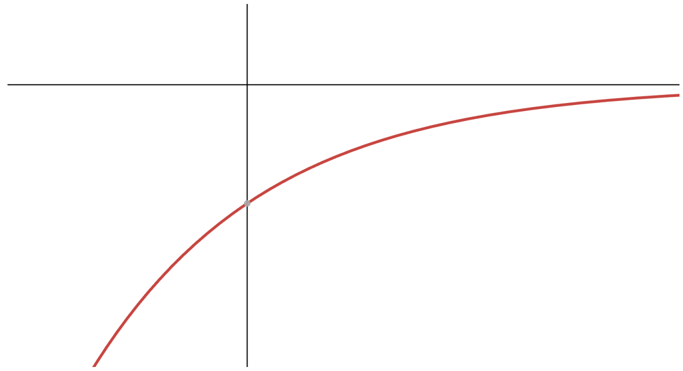
Answer: Since $-3.4e^{-1.27t} = -3.4(e^{-1.27})^t = -3.4(0.281)^t$, the decay rate is 71.9%.

v. What is the continuous growth/decay rate?

Answer: 127%.

vi. Sketch a graph of the function.

Answer: With a y-intercept of -3.4.



2. The following functions give the populations of four towns with time t in years.

   i $P = 600(1.12)^t$

   ii $P = 1,000(1.03)^t$

   iii $P = 200(1.08)^t$

   iv $P = 900(0.90)^t$

   (a) Which town has the largest percent growth? What is the percent growth rate?

   Answer: Town i with a growth rate of 12%.

   (b) Which town ha the largest initial population? What is that initial population?

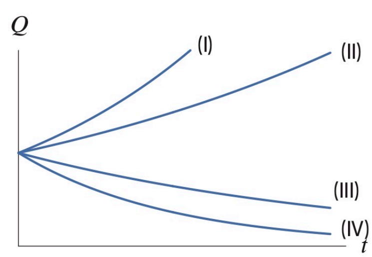
   Answer: Town ii with an initial population of 1,000.

   (c) Are there any towns decreasing in size? If so, which one(s) and by what percent are they decreasing?

   Answer: Town iv, which is decreasing at a rate of 10% per year.

3. The graph in the figure below shows $Q = 50(1.2)^t$, $Q = 50(0.6)^t$, $Q = 50(0.8)^t$, and $Q = 50(1.4)^t$. Match each formula to a graph.

   Answer: I is the largest increase, which is $Q = 50(1.4)^t$. II is the smaller increase, which is $Q = 50(1.2)^t$. III is the slower decay rate, which is $Q = 50(0.8)^t$, with a decay rate of 20%, and IV is the faster decay rate, which is $Q = 50(0.6)^t$, with a decay rate of 40%.



4. The figure below shows graphs of several cities' population against time. Match each of the following descriptions to a graph and write a description to match each of the remaining graphs.

   (a) The population increased by 5% per year.

   Answer: II

   (b) The population increased by 8% per year.

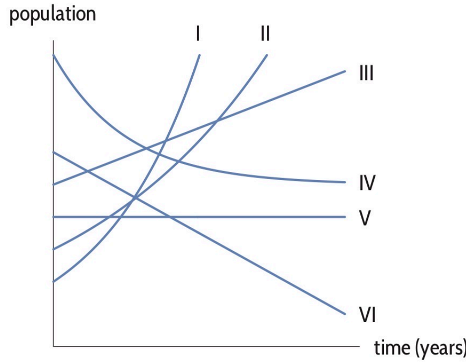
   Answer: I

   (c) The population increased by 5,000 people per year.

   Answer: III

   (d) The population was stable.

   Answer: V. For the remaining graphs, we have for IV that it is decaying at, say, 6% per year and for VI, we could day it is decaying by, say, 8,000 people per year.



5. The exponential functions graphed below have b, d, q positive.

   (a) Which of the constants a, c, and p must be positive? Explain.

   Answer: They must all be positive, because they represent the initial value of each function, and each function intersects the vertical axis at a positive value.

   (b) Which of the constants a, b, c, d, p, and q must be between 0 and 1? Explain.

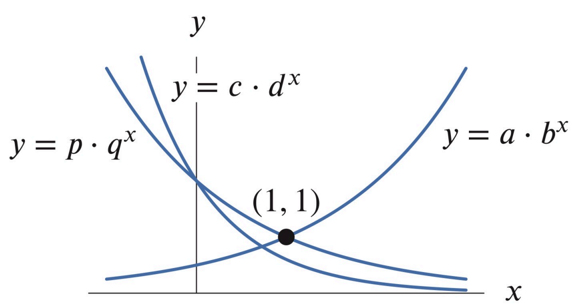
   Answer: a is between 0 and 1, because it represents the initial value of the function which is positive but less than 1. Also, q and d must be between 0 and 1, because they represent the rate of growth/decay of their respective functions, and both of those functions are decaying.

   (c) Which two of the constants a, b, c, d, p, and q must be equal?

   Answer: p and c must be equal, since they represent the same initial value on the vertical axis.

   (d) What information about the constants a and b does the point $(1, 1)$ provide?

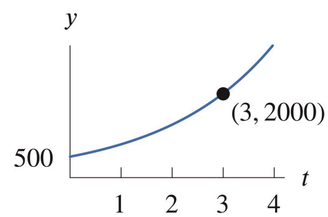
   Answer: If we substitute in 1 for both x and y, we see that $a \cdot b = 1$, so that means that a and b are reciprocals of one another.

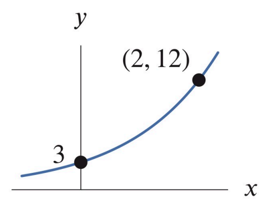


6. Give a possible formula for the functions in the figure below:

   Answer: For the first, we see that the initial value is 500, so the function is of the form $y = 500 \cdot b^t$. Also, when we put $t = 3$, we get $y = 2000$, so $2000 = 500 \cdot b^3$. Therefore $b = 1.587$. So, the function is $y = 500(1.587)^t$.

   For the second, we have $y = 3 \cdot b^x$ and when $x = 2$, $y = 12$. Therefore, $12 = 3 \cdot b^2$, so that $b = 2$. The function is therefore $y = 3 \cdot 2^x$.

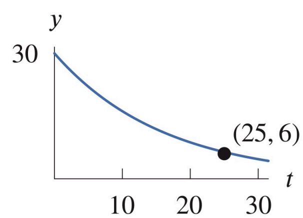


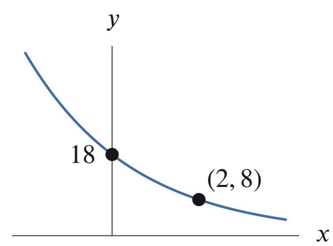


7. Give a possible formula for the functions in the figure below:

   Answer: For the first one, we see that $y = 30 \cdot b^t$. When $t = 25$, $y = 6$. Therefore, $6 = 30 \cdot b^{25}$, so that $b = 0.938$. The function is $y = 30(0.938)^t$.

   For the second, we see that $y = 18 \cdot b^x$ and when $x = 2$, $y = 8$. Therefore, $8 = 18 \cdot b^2$, so that $b = 0.667$. The function is $y = 18(0.667)^x$.





8. Write a possible formula that describes each of the following situations.

   (a) An air-freshener starts with 30 grams of material that evaporates at 2 grams per day.

   Answer: $M = 30 - 2t$.

   (b) An air-freshener starts with 30 grams of material that evaporates by 2% per day.

   Answer: $M = 30(0.98)^t$.

   (c) A town has a population of 1,000 people at time $t = 0$, and the population increases by 50 people every year.

   Answer: $P = 1000 + 50t$.

   (d) A town has a population of 1,000 people at time $t = 0$, and the population increases by 5% people every year.

   Answer: $P = 1000(1.05)^t$.

(e) The gross domestic product, G, of Switzerland was 685.4 billion dollars in 2013 and increases by 2% per year.

Answer: $G = 685.4(1.02)^t$.

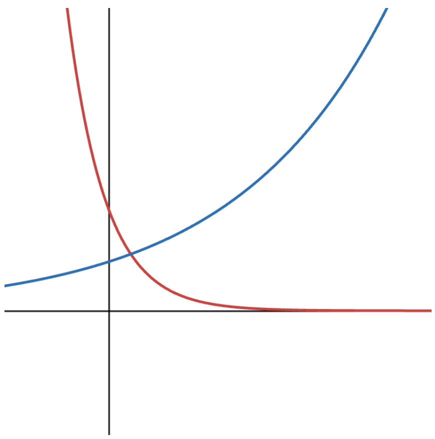
(f) The gross domestic product, G, of Switzerland was 685.4 billion dollars in 2013 and increases by 7 billion dollars per year.

Answer: $G = 685.4 + 7t$.

9. For each of the following exponential functions, sketch a quick graph to determine whether they intersect for some positive time value and, if they do, find the coordinates of that intersection.

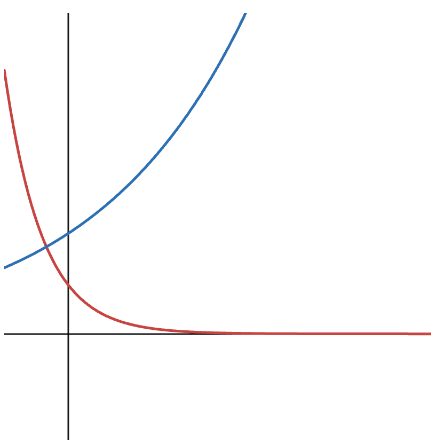
(a) $s = 4.7(0.57)^t$ and $s = 2.3(1.15)^t$.

Answer: According to the graph below, there is an intersection for positive time values. To find it, we solve $4.7(0.57)^t = 2.3(1.15)^t \rightarrow \frac{4.7}{2.3} = \frac{1.15^t}{0.57^t} \rightarrow 2.043 = 2.017^t \rightarrow \ln(2.043) = t \cdot \ln(2.017) \rightarrow t = \frac{\ln(2.043)}{\ln(2.017)} \approx 1.0182$. To find the s value, we plug this t value into either equation to get $s = 2.652$.



(b) $s = 2.3(0.57)^t$ and $s = 4.7(1.15)^t$.

Answer: According to the graph below, there is no intersection for positive time values.



10. A town has a population of 1,000 people at time $t = 0$, and the population increases by 5% people every year. Find the amount of time it will take for the population to double.

Answer: The formula that models this behavior is $P = 1000(1.05)^t$. We want see how long it will take for the population to get to $2,000$. So, we solve $2000 = 1000(1.05)^t \rightarrow 2 = (1.05)^t \rightarrow \ln 2 = t \cdot \ln 1.05 \rightarrow t = \frac{\ln 2}{\ln 1.05} \approx 14.207$ years.