Problems for Section 1.2

■ For Problems 1–4, find an equation for the line that passes through the given points.

1. $(0, 2)$ and $(2, 3)$
2. $(0, 0)$ and $(1, 1)$
3. $(-2, 1)$ and $(2, 3)$
4. $(4, 5)$ and $(2, -1)$

■ For Problems 5–8, determine the slope and the y-intercept of the line whose equation is given.

5. $7y + 12x - 2 = 0$
6. $3x + 2y = 8$
7. $12x = 6y + 4$ s
8. $-4y + 2x + 8 = 0$

9. Figure 1.22 shows four lines given by equation $y = b + mx$. Match the lines to the conditions on the parameters m and b.

(a) $m > 0, b > 0$ **(b)** $m < 0, b > 0$
(c) $m > 0, b < 0$ **(d)** $m < 0, b < 0$

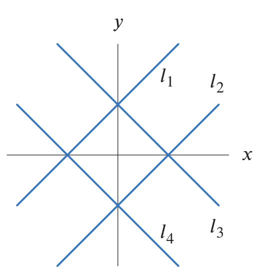

Figure 1.22

10. **(a)** Which two lines in Figure 1.23 have the same slope? Of these two lines, which has the larger y-intercept?
(b) Which two lines have the same y-intercept? Of these two lines, which has the larger slope?

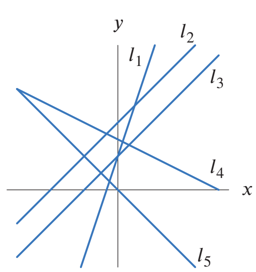

Figure 1.23

11. A city's population was 28,600 in the year 2015 and is growing by 750 people a year.
(a) Give a formula for the city's population, P, as a function of the number of years, t, since 2015.
(b) What is the population predicted to be in 2020?
(c) When is the population expected to reach 35,000?

12. A car rental company charges a daily fee of \$35 plus \$0.20 per mile driven. Find a formula for the daily charge, C, in dollars, as a function of the number of miles, m, driven that day.

13. A company rents cars at \$40 a day and 15 cents a mile. Its competitor's cars are \$50 a day and 10 cents a mile.
(a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled.
(b) On the same axes, graph both functions.
(c) How should you decide which company is cheaper?

14. A phone company charges \$14.99 a month and 10 cents for every minute above 120 minutes. Write an expression for the monthly phone charge, P, in dollars, as a function of the number of minutes, m, used that month.

15. Figure 1.24 shows the distance from home, in miles, of a person on a 5-hour trip.
(a) Estimate the vertical intercept. Give units and interpret it in terms of distance from home.
(b) Estimate the slope of this linear function. Give units, and interpret it in terms of distance from home.
(c) Give a formula for distance, D, from home as a function of time, t in hours.

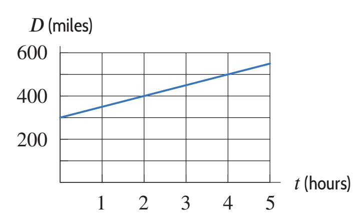

Figure 1.24

16. Which of the following tables could represent linear functions?

(a)

x	0	1	2	3
y	27	25	23	21

(b)

t	15	20	25	30
s	62	72	82	92

(c)

u	1	2	3	4
w	5	10	18	28

17. For each table in Problem 16 that could represent a linear function, find a formula for that function.

18. Annual revenue R from McDonald's restaurants worldwide between 2013 and 2015 can be estimated by $R = 28.1 - 1.35t$, where R is in billion dollars and t is in years since January 1, 2013.[15]

(a) What is the slope of this function? Include units. Interpret the slope in terms of McDonald's revenue.
(b) What is the vertical intercept of this function? Include units. Interpret the vertical intercept in terms of McDonald's revenue.
(c) What annual revenue does the function predict for 2018?
(d) If revenue continues to decline at the same rate, when is annual revenue predicted to fall to 22 billion dollars?

■ Problems 19–21 use Figure 1.25 showing how the quantity, Q, of grass (kg/hectare) in different parts of Namibia depended on the average annual rainfall, r, (mm), in two different years.[16]

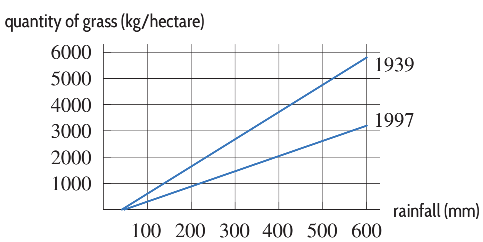

Figure 1.25

19. **(a)** For 1939, find the slope of the line, including units.
(b) Interpret the slope in this context.
(c) Find the equation of the line.

20. **(a)** For 1997, find the slope of the line, including units.
(b) Interpret the slope in this context.
(c) Find the equation of the line.

21. Which of the two functions in Figure 1.25 has the larger difference quotient $\Delta Q/\Delta r$? What does this tell us about grass in Namibia?

22. US imports of crude oil and petroleum have been increasing.[17] There have been many ups and downs, but the general trend is shown by the line in Figure 1.26.

(a) Find the slope of the line. Include its units of measurement.
(b) Write an equation for the line. Define your variables, including their units.
(c) Assuming the trend continues, when does the linear model predict imports will reach 18 million barrels per day? Do you think this is a reliable prediction? Give reasons.

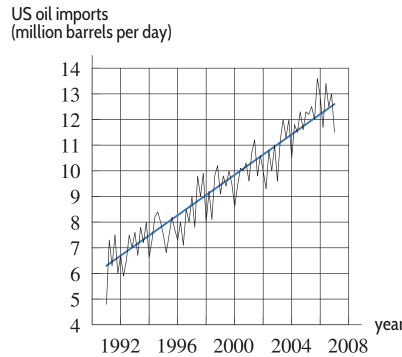

Figure 1.26

23. A company's pricing schedule in Table 1.5 is designed to encourage large orders. (A gross is 12 dozen.) Find a formula for:

(a) q as a linear function of p.
(b) p as a linear function of q.

Table 1.5

q (order size, gross)	3	4	5	6
p (price/dozen)	15	12	9	6

24. World milk production rose at an approximately constant rate between 2000 and 2012.[18] See Figure 1.27.

(a) Estimate the vertical intercept and interpret it in terms of milk production.
(b) Estimate the slope and interpret it in terms of milk production.
(c) Give an approximate formula for milk production, M, as a function of t.

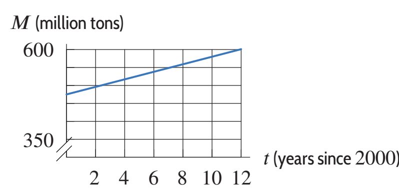

Figure 1.27

[15] Based on McDonald's Annual Report 2015, corporate.mcdonalds.com/mcd/investors/financial-information/annual-report.html, accessed September 15, 2016.
[16] David Ward and Ben T. Ngairorue, "Are Namibia's Grasslands Desertifying?", *Journal of Range Management* 53, 2000, 138–144.
[17] http://www.theoildrum.com/node/2767. Accessed May 2015.
[18] www.dairyco.org.uk/library/market-information/datum/world-milk-production.aspx, accessed September 15, 2016.

25. Marmots are large squirrels that hibernate in the winter and come out in the spring. Figure 1.28 shows the date (days after Jan 1) that they are first sighted each year in Colorado as a function of the average minimum daily temperature for that year.[19]

(a) Find the slope of the line, including units.
(b) What does the sign of the slope tell you about marmots?
(c) Use the slope to determine how much difference 6°C warming makes to the date of first appearance of a marmot.
(d) Find the equation of the line.

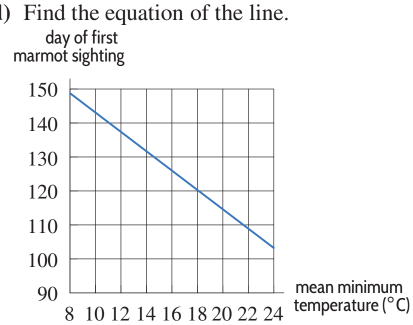

Figure 1.28

26. In Colorado spring has arrived when the bluebell first flowers. Figure 1.29 shows the date (days after Jan 1) that the first flower is sighted in one location as a function of the first date (days after Jan 1) of bare (snow-free) ground.[20]

(a) If the first date of bare ground is 140, how many days later is the first bluebell flower sighted?
(b) Find the slope of the line, including units.
(c) What does the sign of the slope tell you about bluebells?
(d) Find the equation of the line.

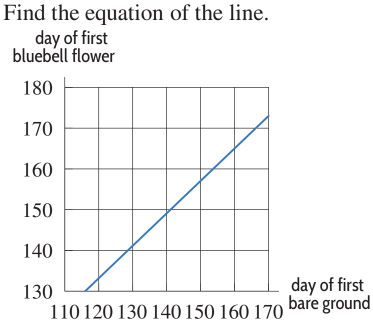

Figure 1.29

27. On March 5, 2015, Capracotta, Italy, received 256 cm (100.787 inches) of snow in 18 hours.[21]

(a) Assuming the snow fell at a constant rate and there were already 100 cm of snow on the ground, find a formula for $f(t)$, in cm, for the depth of snow as a function of t hours since the snowfall began on March 5.
(b) What are the domain and range of f?

28. The approximate percentage of people, P, below the poverty level in the US[22] is given in Table 1.6.

(a) Find a formula for the percentage in poverty as a linear function of time in years since 2007.
(b) Use the formula to predict the percentage in poverty in 2013.
(c) What is the difference between the prediction and the actual percentage, 14.5%?

Table 1.6

Year (since 2007)	0	1	2	3
P (percentage)	12.5	13.4	14.3	15.2

29. World grain production was 1241 million tons in 1975 and 2048 million tons in 2005, and has been increasing at an approximately constant rate.[23]

(a) Find a linear function for world grain production, P, in million tons, as a function of t, the number of years since 1975.
(b) Using units, interpret the slope in terms of grain production.
(c) Using units, interpret the vertical intercept in terms of grain production.
(d) According to the linear model, what is the predicted world grain production in 2015?
(e) According to the linear model, when is grain production predicted to reach 2500 million tons?

30. Annual sales of music compact discs (CDs) have declined since 2000. Sales were 942.5 million in 2000 and 384.7 million in 2008.[24]

(a) Find a formula for annual sales, S, in millions of music CDs, as a linear function of the number of years, t, since 2000.
(b) Give units for and interpret the slope and the vertical intercept of this function.
(c) Use the formula to predict music CD sales in 2012.

[19] David W. Inouye, Billy Barr, Kenneth B. Armitage, and Brian D. Inouye, "Climate Change is Affecting Altitudinal Migrants and Hibernating Species", *PNAS* 97, 2000, 1630–1633.
[20] David W. Inouye, Billy Barr, Kenneth B. Armitage, and Brian D. Inouye, "Climate Change is Affecting Altitudinal Migrants and Hibernating Species", *PNAS* 97, 2000, 1630–1633.
[21] http://iceagenow.info/2015/03/official-italy-captures-world-one-day-snowfall-record/
[22] Data based on: www.huffingtonpost.com/2014/09/16/poverty-household-income_n_5828974.html, accessed September 15, 2016.
[23] The Worldwatch Institute, *Vital Signs 2007-2008* (New York: W.W. Norton & Company, 2007), p. 21.
[24] *The World Almanac and Book of Facts 2008* (New York).

31. A \$25,000 vehicle depreciates \$2000 a year as it ages. Repair costs are \$1500 per year.

(a) Write formulas for each of the two linear functions at time t, value, $V(t)$, and repair costs to date, $C(t)$. Graph them.
(b) One strategy is to replace a vehicle when the total cost of repairs is equal to the current value. Find this time.
(c) Another strategy is to replace the vehicle when the value of the vehicle is some percent of the original value. Find the time when the value is 6%.

32. Search and rescue teams work to find lost hikers. Members of the search team separate and walk parallel to one another through the area to be searched. Table 1.7 shows the percent, P, of lost individuals found for various separation distances, d, of the searchers.[25]

(a) Explain how you know that the percent found, P, could be a linear function of separation distance, d.
(b) Find P as a linear function of d.
(c) What is the slope of the function? Give units and interpret the answer.
(d) What are the vertical and horizontal intercepts of the function? Give units and interpret the answers.

Table 1.7

Separation distance d (ft)	20	40	60	80	100
Approximate percent found, P	90	80	70	60	50

33. In a Washington town, the charge for commercial waste collection is \$694.55 for 5 tons and \$1098.32 for 8 tons of waste.

(a) Find a linear formula for the cost, C, of waste collection as a function of the weight, w, in tons.
(b) What is the slope of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.
(c) What is the vertical intercept of the line found in part (a)? Give units and interpret your answer in terms of the cost of waste collection.

34. The number of species of coastal dune plants in Australia decreases as the latitude, in °S, increases. There are 34 species at 11°S and 26 species at 44°S.[26]

(a) Find a formula for the number, N, of species of coastal dune plants in Australia as a linear function of the latitude, l, in °S.
(b) Give units for and interpret the slope and the vertical intercept of this function.
(c) Graph this function between $l = 11$°S and $l = 44$°S. (Australia lies entirely within these latitudes.)

35. Table 1.8 gives the required standard weight, w, in kilograms, of American soldiers, aged between 21 and 27, for height, h, in centimeters.[27]

(a) How do you know that the data in this table could represent a linear function?
(b) Find weight, w, as a linear function of height, h. What is the slope of the line? What are the units for the slope?
(c) Find height, h, as a linear function of weight, w. What is the slope of the line? What are the units for the slope?

Table 1.8

h (cm)	172	176	180	184	188	192	196
w (kg)	79.7	82.4	85.1	87.8	90.5	93.2	95.9

■ Problems 36–41 concern the maximum heart rate (MHR), which is the maximum number of times a person's heart can safely beat in one minute. If MHR is in beats per minute and a is age in years, the formulas used to estimate MHR are

$$\text{For females: MHR} = 226 - a,$$

$$\text{For males: MHR} = 220 - a.$$

36. Which of the following is the correct statement?

(a) As you age, your maximum heart rate decreases by one beat per year.
(b) As you age, your maximum heart rate decreases by one beat per minute.
(c) As you age, your maximum heart rate decreases by one beat per minute per year.

37. Which of the following is the correct statement for a male and female of the same age?

(a) Their maximum heart rates are the same.
(b) The male's maximum heart rate exceeds the female's.
(c) The female's maximum heart rate exceeds the male's.

38. What can be said about the ages of a male and a female with the same maximum heart rate?

[25] J. Wartes, *An Experimental Analysis of Grid Sweep Searching* (Explorer Search and Rescue, Western Region, 1974).
[26] M. L. Rosenzweig, *Species Diversity in Space and Time* (Cambridge: Cambridge University Press, 1995), p. 292.
[27] Adapted from usmilitary.about.com, accessed March 29, 2015.

39. Recently[28] it has been suggested that a more accurate predictor of MHR for both males and females is given by

$$\text{MHR} = 208 - 0.7a.$$

(a) At what age do the old and new formulas give the same MHR for females? For males?

(b) Which of the following is true?

(i) The new formula predicts a higher MHR for young people and a lower MHR for older people than the old formula.

(ii) The new formula predicts a lower MHR for young people and a higher MHR for older people than the old formula.

(c) When testing for heart disease, doctors ask patients to walk on a treadmill while the speed and incline are gradually increased until their heart rates reach 85 percent of the MHR. For a 65-year-old male, what is the difference in beats per minute between the heart rate reached if the old formula is used and the heart rate reached if the new formula is used?

40. Experiments[29] suggest that the female MHR decreases by 12 beats per minute by age 21, and by 19 beats per minute by age 33. Is this consistent with MHR being approximately linear with age?

41. Experiments[30] suggest that the male MHR decreases by 9 beats per minute by age 21, and by 26 beats per minute by age 33. Is this consistent with MHR being approximately linear with age?

42. Let y be the percent increase in annual US national production during a year when the unemployment rate changes by u percent. (For example, $u = 2$ if unemployment increases from 4% to 6%.) Okun's law states that

$$y = 3.5 - 2u.$$

(a) What is the meaning of the number 3.5 in Okun's law?

(b) What is the effect on national production of a year when unemployment rises from 5% to 8%?

(c) What change in the unemployment rate corresponds to a year when production is the same as the year before?

(d) What is the meaning of the coefficient -2 in Okun's law?

43. An Australian[31] study found that, if other factors are constant (education, experience, etc.), taller people receive higher wages for the same work. The study reported a "height premium" for men of 3% of the hourly wage for a 10 cm increase in height; for women the height premium reported was 2%. We assume that hourly wages are a linear function of height. The slope is given by the height premium at the average hourly wage for that gender, AU\$29.40 for men and AU\$24.78 for women.

(a) The average hourly wage[32] for a 178 cm Australian man is AU\$29.40. Express the average hourly wage of an Australian man as a function of his height, x cm.

(b) The average hourly wage for a 164 cm Australian woman is AU\$24.78. Express the average hourly wage of an Australian woman as a function of her height, y cm.

(c) What is the difference in average hourly wages between men and women of height 178 cm?

(d) Is there a height for which men and women are predicted to have the same wage? If so, what is it?

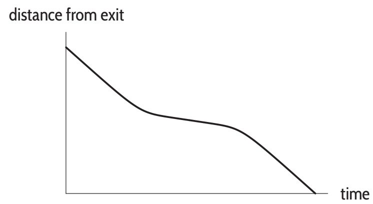

Figure 1.14

40. See Figure 1.15.

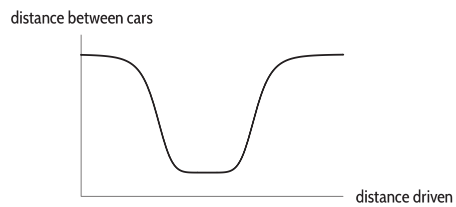

Figure 1.15

Solutions for Section 1.2

1. The slope is $(3-2)/(2-0) = 1/2$. So the equation of the line is $y = (1/2)x + 2$.

2. The slope is $(1-0)/(1-0) = 1$. So the equation of the line is $y = x$.

3. The slope is

$$\text{Slope } = \frac{3-1}{2-(-2)} = \frac{2}{4} = \frac{1}{2}.$$

Now we know that $y = (1/2)x + b$. Using the point $(-2, 1)$, we have $1 = -2/2 + b$, which yields $b = 2$. Thus, the equation of the line is $y = (1/2)x + 2$.

4. The slope is

$$\text{Slope } = \frac{-1-5}{2-4} = \frac{-6}{-2} = 3.$$

Substituting $m = 3$ and the point $(4, 5)$ into the equation of the line, $y = b + mx$, we have

$$\begin{aligned} 5 &= b + 3 \cdot 4 \\ 5 &= b + 12 \\ b &= -7. \end{aligned}$$

The equation of the line is $y = -7 + 3x$.

5. Rewriting the equation as

$$y = -\frac{12}{7}x + \frac{2}{7}$$

shows that the line has slope $-12/7$ and vertical intercept $2/7$.

6. Rewriting the equation as

$$y = 4 - \frac{3}{2}x$$

shows that the line has slope $-3/2$ and vertical intercept 4.

7. Rewriting the equation of the line as

$$y = \frac{12}{6}x - \frac{4}{6}$$
$$y = 2x - \frac{2}{3},$$

we see that the line has slope 2 and vertical intercept $-2/3$.
s

8. Rewriting the equation of the line as

$$-y = \frac{-2}{4}x - 2$$
$$y = \frac{1}{2}x + 2,$$

we see the line has slope $1/2$ and vertical intercept 2.

9. The slope is positive for lines l_1 and l_2 and negative for lines l_3 and l_4. The y-intercept is positive for lines l_1 and l_3 and negative for lines l_2 and l_4. Thus, the lines match up as follows:

(a) l_1
(b) l_3
(c) l_2
(d) l_4

10. **(a)** Lines l_2 and l_3 are parallel and thus have the same slope. Of these two, line l_2 has a larger y-intercept.
(b) Lines l_1 and l_3 have the same y-intercept. Of these two, line l_1 has a larger slope.

11. **(a)** The slope is constant at 750 people per year so this is a linear function. The vertical intercept (at $t = 0$) is 28,600. We have

$$P = 28{,}600 + 750t.$$

(b) The population in 2020, when $t = 5$, is predicted to be

$$P = 28{,}600 + 750 \cdot 5 = 32{,}350.$$

(c) We use $P = 35{,}000$ and solve for t:

$$\begin{aligned} P &= 28{,}600 + 750t \\ 35{,}000 &= 28{,}600 + 750t \\ 6{,}400 &= 750t \\ t &= 8.53. \end{aligned}$$

The population will reach 35,000 people in about 8.53 years, or during the year 2023.

12. This is a linear function with vertical intercept 35 and slope 0.20. The formula for the monthly charge is

$$C = 35 + 0.20m.$$

13. **(a)** The first company's price for a day's rental with m miles on it is $C_1(m) = 40 + 0.15m$. Its competitor's price for a day's rental with m miles on it is $C_2(m) = 50 + 0.10m$.
(b) See Figure 1.16.

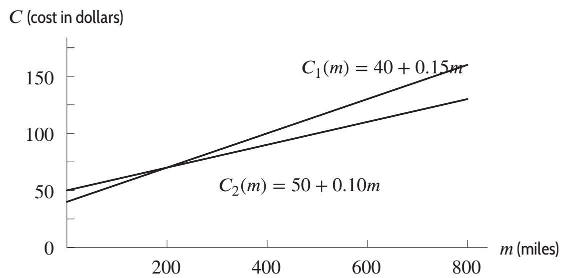

Figure 1.16

(c) To find which company is cheaper, we need to determine where the two lines intersect. We let $C_1 = C_2$, and thus

$$\begin{aligned} 40 + 0.15m &= 50 + 0.10m \\ 0.05m &= 10 \\ m &= 200. \end{aligned}$$

If you are going more than 200 miles a day, the competitor is cheaper. If you are going less than 200 miles a day, the first company is cheaper.

14. This is a linear function with two parts to the formula:
For $0 \le m \le 120$, we pay only the \$14.99, so

$$P = 14.99.$$

For $120 < m$, the 10-cent charge is only on the minutes above 120, that is $m - 120$ minutes:

$$P = 14.99 + 0.1(m - 120).$$

15. **(a)** The vertical intercept appears to be about 300 miles. When this trip started (at $t = 0$) the person was 300 miles from home.

(b) The slope appears to be about $(550 - 300)/5 = 50$ miles per hour. This person is moving away from home at a rate of about 50 miles per hour.

(c) We have $D = 300 + 50t$.

16. **(a)** On the interval from 0 to 1 the value of y decreases by 2. On the interval from 1 to 2 the value of y decreases by 2. And on the interval from 2 to 3 the value of y decreases by 2. Thus, the function has a constant rate of change and it could therefore be linear.

(b) On the interval from 15 to 20 the value of s increases by 10. On the interval from 20 to 25 the value of s increases by 10. And on the interval from 25 to 30 the value of s increases by 10. Thus, the function has a constant rate of change and could be linear.

(c) On the interval from 1 to 2 the value of w increases by 5. On the interval from 2 to 3 the value of w increases by 8. Thus, we see that the slope of the function is not constant and so the function is not linear.

17. For the function given by table (a), we know that the slope is

$$\text{slope} = \frac{27 - 25}{0 - 1} = -2.$$

We also know that at $x = 0$ we have $y = 27$. Thus we know that the vertical intercept is 27. The formula for the function is

$$y = -2x + 27.$$

For the function in table (b), we know that the slope is

$$\text{slope} = \frac{72 - 62}{20 - 15} = \frac{10}{5} = 2.$$

Thus, we know that the function will take on the form

$$s = 2t + b.$$

Substituting in the coordinates (15, 62) we get

$$\begin{aligned} s &= 2t + b \\ 62 &= 2(15) + b \\ &= 30 + b \\ 32 &= b. \end{aligned}$$

Thus, a formula for the function would be

$$s = 2t + 32.$$

18. **(a)** The slope is -1.35 billion dollars per year. McDonald's revenue is decreasing at a rate of 1.35 billion dollars per year.
(b) The vertical intercept is 28.1 billion dollars. In 2013, McDonald's revenue was 28.1 billion dollars.
(c) Substituting $t = 5$, we have $R = 28.1 - 1.35 \cdot 5 = 21.35$ billion dollars.
(d) We substitute $R = 22$ and solve for t:

$$\begin{aligned} 28.1 - 1.35t &= 22 \\ -1.35t &= -6.1 \\ t &= \frac{6.1}{1.35} \approx 4.52 \text{ years.} \end{aligned}$$

The function predicts that annual revenue will fall to 22 billion dollars in 2017.

19. **(a)** Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 600$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 5800$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{5800 - 600}{600 - 100} = 10.4 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 10.4 kg of grass per hectare.
(c) Using the slope, we see that the equation has the form

$$Q = b + 10.4r.$$

Substituting $r = 100$ and $Q = 600$ we can solve for b.

$$\begin{aligned} b + 10.4(100) &= 600 \\ b &= -440. \end{aligned}$$

The equation of the line is

$$Q = -440 + 10.4r.$$

20. **(a)** Reading coordinates from the graph, we see that rainfall $r = 100$ mm corresponds to about $Q = 300$ kg/hectare, and $r = 600$ mm corresponds to about $Q = 3200$ kg/hectare. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta Q}{\Delta r} = \frac{3200 - 300}{600 - 100} = 5.8 \text{ kg/hectare per mm.}$$

(b) Every additional 1 mm of annual rainfall corresponds to an additional 5.8 kg of grass per hectare.
(c) Using the slope, we see that the equation has the form

$$Q = b + 5.8r.$$

Substituting $r = 100$ and $Q = 300$ we can solve for b.

$$\begin{aligned} b + 5.8(100) &= 300 \\ b &= -280. \end{aligned}$$

The equation of the line is

$$Q = -280 + 5.8r.$$

21. The difference quotient $\Delta Q/\Delta r$ equals the slope of the line and represents the increase in the quantity of grass per millimeter of rainfall. We see from the graph that the slope of the line for 1939 is larger than the slope of the line for 1997. Thus, each additional 1 mm of rainfall in 1939 led to a larger increase in the quantity of grass than in 1997.

22. **(a)** We select two points on the line. Using the values (1992, 6.75) and (2006, 12.25) we have

$$\text{Slope} = \frac{12.25 - 6.75}{2006 - 1992 \text{ years}} = 0.4 \ \frac{\text{million barrels per day}}{\text{year}}.$$

(b) With t as the year and $f(t)$ as the quantity of imports in millions of barrels per day, we have

$$f(t) = 6.75 + 0.4(t - 1992).$$

Alternatively, if t as the number of years since 1992 and if $f(t)$ is the quantity of imports in millions of barrels per day

$$f(t) = 6.75 + 0.4t.$$

(c) We have

$$\begin{aligned} 6.75 + 0.4t &= 18 \\ t &= 28.125. \end{aligned}$$

The model predicts imports will reach 18 million barrels a day in the year $1992 + 28.125 = 2020.125$.

This prediction could serve as a guideline, but it is very risky to put too much reliance on a prediction more than ten years into the future, especially since there has been a drop in imports since 2008. Many unexpected events could drastically change the economic environment during that time. This prediction should be used with caution.

23. **(a)** We know that the function for q in terms of p will take on the form

$$q = mp + b.$$

We know that the slope will represent the change in q over the corresponding change in p. Thus

$$m = \text{slope} = \frac{4 - 3}{12 - 15} = \frac{1}{-3} = -\frac{1}{3}.$$

Thus, the function will take on the form

$$q = -\frac{1}{3}p + b.$$

Substituting the values $q = 3$, $p = 15$, we get

$$\begin{aligned} 3 &= -\frac{1}{3}(15) + b \\ 3 &= -5 + b \\ b &= 8. \end{aligned}$$

Thus, the formula for q in terms of p is

$$q = -\frac{1}{3}p + 8.$$

(b) We know that the function for p in terms of q will take on the form

$$p = mq + b.$$

We know that the slope will represent the change in p over the corresponding change in q. Thus

$$m = \text{slope} = \frac{12 - 15}{4 - 3} = -3.$$

Thus, the function will take on the form

$$p = -3q + b.$$

Substituting the values $q = 3$, $p = 15$ again, we get

$$\begin{aligned} 15 &= (-3)(3) + b \\ 15 &= -9 + b \\ b &= 24. \end{aligned}$$

Thus, a formula for p in terms of q is

$$p = -3q + 24.$$

24. **(a)** The vertical intercept appears to be about 480. In 2000, world milk production was about 480 million tons of milk.

(b) World milk production appears to be about 480 at $t = 0$ and about 600 at $t = 12$. We estimate

$$\text{Slope} = \frac{600 - 480}{12 - 0} = 10 \text{ million tons/year.}$$

World milk production increased at a rate of about 10 million tons per year during this 12-year period.

(c) We have $M = 480 + 10t$.

25. **(a)** Let x be the average minimum daily temperature (°C), and let y be the date the marmot is first sighted. Reading coordinates from the graph, we see that temperature $x = 12$°C corresponds to date $y = 137$ days after Jan 1, and $x = 22$°C corresponds to $y = 109$ days. Using a difference quotient, we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{109 - 137}{22 - 12} = -2.8 \,\frac{\text{days}}{{}^\circ\text{C}}.$$

(b) The slope is negative. An increase in the temperatures corresponds to an earlier sighting of a marmot. Marmots come out of hibernation earlier in years with warmer average daily minimum temperature.

(c) We have

$$\Delta y = \text{Slope} \times \Delta x = (-2.8)(6) = 16.8 \text{ days.}$$

If temperatures are 6°C higher, then marmots come out of hibernations about 17 days earlier.

(d) Using the slope, we see that the equation has the form

$$y = b - 2.8x.$$

Substituting $x = 12$ and $y = 137$ we solve for b.

$$\begin{aligned} b - 2.8(12) &= 137 \\ b &= 171. \end{aligned}$$

The equation of the line is

$$y = 171 - 2.8x.$$

26. **(a)** If the first date of bare ground is 140, then, according to the figure, the first bluebell flower is sighted about day 150, that is $150 - 140 = 10$ days later.

(b) Let x be the first date of bare ground, and let y be the date the first bluebell flower is sighted. Reading coordinates from the graph, we see that date $x = 130$ days after Jan 1 corresponds to date $y = 142$ days after Jan 1, and $x = 170$ days corresponds to $y = 173$ days. Using a difference quotient we have

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{173 - 142}{170 - 130} = 0.775 \text{ days per day.}$$

(c) The slope is positive, so an increase in the x-variable corresponds to an increase in the y-variable. These variables represents days in the year, and larger values indicate days that are later in the year. Thus if bare ground first occurs later in the year, then bluebells first flower later in the year. Positive slope means that bluebells flower later when the snow cover lasts longer.

(d) Using the slope, we see that the equation has the form

$$y = b + 0.775x.$$

Substituting $x = 130$ and $y = 142$ we can solve for b.

$$\begin{aligned} b + 0.775(130) &= 142 \\ b &= 41.25. \end{aligned}$$

The equation of the line is

$$y = 41.25 + 0.775x.$$

27. **(a)** We have $m = \dfrac{256}{18} = 14.222$ cm per hour. When the snow started, there were 100 cm on the ground, so

$$f(t) = 100 + 14.222t.$$

(b) The domain of f is $0 \le t \le 18$ hours. The range is $100 \le f(t) \le 356$ cm.

28. **(a)** A linear function has the form

$$P = b + mt$$

where t is in years since 2007. The slope is

$$\text{Slope } = \frac{\Delta P}{\Delta t} = 0.9.$$

The function takes the value 12.5 in the year 2007 (when $t = 0$). Thus, the formula is

$$P = 12.5 + 0.9t.$$

(b) In 2013, we have $t = 6$. We predict $P = 12.5 + 0.9 \cdot 6 = 17.9$, that is, there are 17.9% below the poverty level in 2013.

(c) The difference is $17.9 - 14.5 = 3.4\%$; the prediction is too high.

29. **(a)** We want $P = b + mt$, where t is in years since 1975. The slope is

$$\text{Slope } = m = \frac{\Delta P}{\Delta t} = \frac{2048 - 1241}{30 - 0} = 26.9.$$

Since $P = 1241$ when $t = 0$, the vertical intercept is 1241 so

$$P = 1241 + 26.9t.$$

(b) The slope tells us that world grain production has been increasing at a rate of 26.9 million tons per year.

(c) The vertical intercept tells us that world grain production was 1241 million tons in 1975 (when $t = 0$.)

(d) We substitute $t = 40$ to find $P = 1241 + 26.9 \cdot 40 = 2317$ million tons of grain.

(e) Substituting $P = 2500$ and solving for t, we have

$$\begin{aligned} 2500 &= 1241 + 26.9t \\ 1259 &= 26.9t \\ t &= 46.803. \end{aligned}$$

Grain production is predicted to reach 2500 million tons during the year 2021.

30. **(a)** We are given two points: $S = 942.5$ when $t = 0$ and $S = 384.7$ when $t = 8$. We use these two points to find the slope. Since $S = f(t)$, we know the slope is $\Delta S/\Delta t$:

$$m = \frac{\Delta S}{\Delta t} = \frac{384.7 - 942.5}{8 - 0} = -69.7.$$

The vertical intercept (when $t = 0$) is 942.5, so the linear equation is

$$S = 942.5 - 69.7t.$$

(b) The slope is -69.7 million CDs per year. Sales of music CDs were decreasing at a rate of 69.7 million CDs a year. The vertical intercept is 942.5 million CDs and represents the sales in the year 2000.

(c) In the year 2012, we have $t = 12$ and we predict

$$\text{Sales } = 942.5 - 69.7 \cdot 12 = 106.1 \text{ million CDs.}$$

31. **(a)** Since the initial value is \$25,000 and the slope is -2000, the value of the vehicle at time t is

$$V(t) = 25000 - 2000t.$$

The cost of repairs has initial value 0 and slope 1500, so

$$C(t) = 1500t.$$

Figure 1.17 shows the graphs of these two functions.

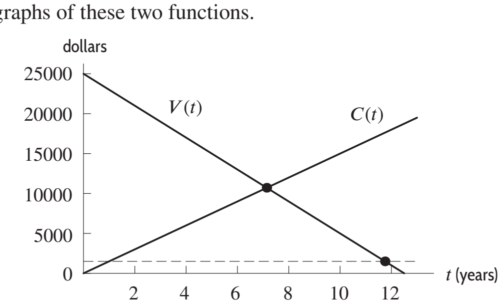

Figure 1.17

(b) The vehicle value equals the repair cost, when $V(t) = C(t)$.

$$\begin{aligned} 25000 - 2000t &= 1500t \\ 25000 &= 3500t \\ 7.143 &= t. \end{aligned}$$

Thus, replace during the eighth year (because the first year ends at $t = 1$). Since 0.143 years is $0.143(12) = 1.7$ months, replacement should take place in the second month of the eighth year.

(c) Since 6% of the orginal value is 1500, the vehicle should be replaced when $V(t) = 1500$.

$$\begin{aligned} 25000 - 2000t &= 1500 \\ 25000 &= 1500 + 2000t \\ 23500 &= 2000t \\ 11.750 &= t. \end{aligned}$$

Thus, in the twelfth year (because the first year ends at $t = 1$). Since 0.75 years is 9 months, replacement should take place at the end of the ninth month of the twelfth year.

32. **(a)** It appears that P is a linear function of d since P decreases by 10 on each interval as d increases by 20.

(b) The slope of the line is

$$m = \frac{\Delta P}{\Delta d} = \frac{-10}{20} = -0.5.$$

We use this slope and the first point in the table to find the equation of the line. We have

$$P = 100 - 0.5d.$$

(c) The slope is -0.5 %/ft. The percent found goes down by 0.5% for every additional foot separating the rescuers.

(d) The vertical intercept is 100%. If the distance separating the rescuers is zero, the percent found is 100%. This makes sense: if the rescuers walk with no distance between them (touching shoulders), they will find everyone.

The horizontal intercept is the value of d when $P = 0$. Substituting $P = 0$, we have

$$\begin{aligned} P &= 100 - 0.5d \\ 0 &= 100 - 0.5d \\ d &= 200. \end{aligned}$$

The horizontal intercept is 200 feet. This is the separation distance between rescuers at which, according to the model, no one is found. This is unreasonable, because even when the searchers are far apart, the search will sometimes be successful. This suggests that at some point, the linear relationship ceases to hold. We have extrapolated too far beyond the given data.

33. **(a)** We find the slope m and intercept b in the linear equation $C = b + mw$. To find the slope m, we use

$$m = \frac{\Delta C}{\Delta w} = \frac{1098.32 - 694.55}{8 - 5} = 134.59 \text{ dollars per ton.}$$

We substitute to find b:

$$\begin{aligned} C &= b + mw \\ 694.55 &= b + (134.59)(5) \\ b &= 21.6 \text{ dollars.} \end{aligned}$$

The linear formula is $C = 21.6 + 134.59w$.

(b) The slope is 134.59 dollars per ton. Each additional ton of waste collected costs \$134.59.

(c) The intercept is \$21.6. The flat fee for waste collection is \$21.6. This is the amount charged even if there is no waste.

34. **(a)** We are given two points: $N = 34$ when $l = 11$ and $N = 26$ when $l = 44$. We use these two points to find the slope. Since $N = f(l)$, we know the slope is $\Delta N/\Delta l$:

$$m = \frac{\Delta N}{\Delta l} = \frac{26 - 34}{44 - 11} = -0.2424.$$

We use the point $l = 11$, $N = 34$ and the slope $m = -0.2424$ to find the vertical intercept:

$$
\begin{aligned}
N &= b + ml \\
34 &= b - 0.2424 \cdot 11 \\
34 &= b - 2.67 \\
b &= 36.67.
\end{aligned}
$$

The equation of the line is

$$N = 36.67 - 0.2424l.$$

(b) The slope is -0.2424 species per degree latitude. In other words, the number of coastal dune plant species decreases by about 0.2424 for every additional degree South in latitude. The vertical intercept is 36.67 species, which represents the number of coastal dune plant species at the equator, if the model (and Australia) extended that far.

(c) See Figure 1.18.

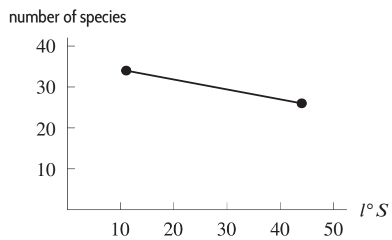

Figure 1.18

35. **(a)** This could be a linear function because w increases by 2.7 as h increases by 4.

(b) We find the slope m and the intercept b in the linear equation $w = b + mh$. We first find the slope m using the first two points in the table. Since we want w to be a function of h, we take

$$m = \frac{\Delta w}{\Delta h} = \frac{82.4 - 79.7}{172 - 168} = 0.68.$$

Substituting the first point and the slope $m = 0.68$ into the linear equation $w = b + mh$, we have $79.7 = b + (0.68)(168)$, so $b = -34.54$. The linear function is

$$w = 0.68h - 34.54.$$

The slope, $m = 0.68$, is in units of kg per cm.

(c) We find the slope and intercept in the linear function $h = b + mw$ using $m = \Delta h/\Delta w$ to obtain the linear function

$$h = 1.47w + 50.79.$$

Alternatively, we could solve the linear equation found in part (b) for h. The slope, $m = 1.47$, has units cm per kg.

36. Both formulas are linear, with slope -1, which means that in one year there is a decrease of one beat per minute, so (c) is correct.

37. If M is the male's maximum heart rate and F is the female's, then

$$
\begin{aligned}
M &= 220 - a \\
F &= 226 - a,
\end{aligned}
$$

so F is 6 larger than M. Thus, female's maximum heart rate exceeds the male's if they are the same age.

38. If f is the age of the female, and m the age of the male, then if they have the same MHR,

$$226 - f = 220 - m,$$

so

$$f - m = 6.$$

Thus the female is 6 years older than the male.

39. **(a)** If old and new formulas give the same MHR for females, we have

$$226 - a = 208 - 0.7a,$$

so

$$a = 60 \text{ years}.$$

For males we need to solve the equation

$$220 - a = 208 - 0.7a,$$

so

$$a = 40 \text{ years}.$$

(b) The old formula starts at either 226 or 220 and decreases. The new formula starts at 208 and decreases slower than the old formula. Thus, the new formula predicts a lower MHR for young people and a higher MHR for older people, so the answer is (ii).

(c) Under the old formula,

$$\text{Heart rate reached } = 0.85(220 - 65) = 131.75 \text{ beats/minute},$$

whereas under the new formula,

$$\text{Heart rate reached } = 0.85(208 - 0.7 \cdot 65) = 138.125 \text{ beats/minute}.$$

The difference is $138.125 - 131.75 = 6.375$ beats/minute more under the new formula.

40. If the MHR is linear with age then the rate of change, in beats per minute per year, is approximately the same at any age. Between 0 and 21 years,

$$\text{Rate of change } = -\frac{12}{21} = -0.571 \text{ beats per minute per year},$$

whereas between 0 and 33 years,

$$\text{Rate of change } = -\frac{19}{33} = -0.576 \text{ beats per minute per year}.$$

These are approximately the same, so they are consistent with MHR being approximately linear with age.

41. If the MHR is linear with age then the slope, in beats per minute per year, would be approximately the same at any age. Between 0 and 21 years,

$$\text{Rate of change } = -\frac{9}{21} = -0.429 \text{ beats per minute per year},$$

whereas between 0 and 33 years,

$$\text{Rate of change } = -\frac{26}{33} = -0.788 \text{ beats per minute per year}.$$

These are not approximately the same, so they are not consistent with MHR being approximately linear with age.

42. **(a)** We have $y = 3.5$ when $u = 0$, which occurs when the unemployment rate does not change. When the unemployment rate is constant, Okun'a law states that US production increases by 3.5% annually.

(b) When unemployment rises from 5% to 8% we have $u = 3$ and therefore

$$y = 3.5 - 2u = 3.5 - 2 \cdot 3 = -2.5.$$

National production for the year decreases by 2.5%.

(c) If annual production does not change, then $y = 0$. Hence $0 = 3.5 - 2u$ and so $u = 1.75$. There is no change in annual production if the unemployment rate goes up 1.75%.

(d) The coefficient -2 is the slope $\Delta y/\Delta u$. Every 1% increase in the unemployment rate during the course of a year results in an additional 2% decrease in annual production for the year.

43. **(a)** The average hourly wage for men is AU\$29.40 and the height premium is 3%, so

$$\text{Slope of men's line } = \frac{(3\%)\ \text{AU\$29.40}}{10 \text{ cm}} = 0.0882 \text{ AU\$/cm}.$$

Since the wage is AU\$29.40 for height $x = 178$, we have

$$\text{Men's wage } = 0.0882(x - 178) + 29.40$$

so

$$\begin{aligned}\text{Men's wage } &= 29.40 + 0.0882x - 0.0882 \cdot 178\\ \text{Men's wage } &= 13.70 + 0.0882x.\end{aligned}$$

(b) The average hourly wage for women is AU\$24.78 and the height premium is 2%, so

$$\text{Slope of women's line } = \frac{(2\%)\ \text{AU\$24.78}}{10 \text{ cm}} = 0.0496 \text{ AU\$/cm}.$$

Since the wage is AU\$24.78 for height $y = 164$, we have

$$\text{Women's wage } = 0.0496(y - 164) + 24.78$$

so

$$\begin{aligned}\text{Women's wage } &= 24.78 + 0.0496y - 0.0496 \cdot 164\\ \text{Women's wage } &= 16.65 + 0.0496y.\end{aligned}$$

(c) For $x = 178$, we are given that

$$\text{Men's average wage } = \text{AU\$29.40}.$$

For $y = 178$, we find

$$\text{Women's average wage } = 16.65 + 0.0496 \cdot 178 = \text{AU\$25.48}.$$

Thus the wage difference is $29.40 - 25.48 =$AU\$3.92.

(d) The line representing men's wages has vertical intercept 13.70 and slope 0.0082; the line representing women's wages has vertical intercept 16.65 and slope 0.0496. Since the vertical intercept of men's wages is lower but the slope is larger, the two lines intersect. Thus, the model predicts there is a height making the two average hourly wages equal. For men and women of the same height, $x = y$. If the average hourly wages are the same

$$\begin{aligned}\text{Men's wage } &= \text{ Women's wage}\\ 13.70 + 0.0882x &= 16.65 + 0.0496x\\ x &= \frac{16.65 - 13.70}{0.0882 - 0.0496} = 76.425 \text{ cm}.\end{aligned}$$

However, this height 76.425 cm, or about 30 inches, or 2 foot 6 inches, is an unrealistic height. Thus for all realistic heights, men's average hourly wage is predicted to be larger than women's average hourly wage.

Solutions for Section 1.3

1. The graph shows a concave down function.

2. The graph shows a concave up function.

3. The graph is concave up.

4. This graph is neither concave up or down.

5. As t increases w decreases, so the function is decreasing. The rate at which w is decreasing is itself decreasing: as t goes from 0 to 4, w decreases by 42, but as t goes from 4 to 8, w decreases by 36. Thus, the function is concave up.