Quiz #3—Solutions

Mathematics 140—Applied Calculus

Dr. Peratt

Directions: Read the following short articles (available on the course web page) and answer the questions below.

1. From "A Prayer for Archimedes," Science News, October 3, 2007:

   (a) Behind the prayer written on the parchment, there were how many texts that appear in no other copy of Archimedes' works?

   Answer: Two manuscripts were found that appear in no other copy of Archimedes' work.

   (b) What was the title of the one that had special historical significance?

   Answer: "The Method."

   (c) What problem was Archimedes trying to solve in this manuscript?

   Answer: He was attempting to compute the areas and volumes of objects with curved surfaces.

   (d) Briefly describe the difference between *potential infinity* and *actual infinity*, including which one was used to approximately quantities arbitrarily closely, but not calculate them exactly?

   Answer: "Potentially infinite" refers to an object or quantity that can be made as large as one wishes, with no limit (like an extendable line segment). "Actually infinite" refers to an object or quantity that is literally infinitely long as it is, like a line.

   As mentioned in the article, exactly calculating some of the areas which interested Archimedes "would require an actual infinity of triangles. Instead, he just said that you can make the approximation as good as you like, so he was sticking with potential infinity."

   This relates to the issue raised in class about $0.999\ldots$ *exactly* equaling 1. If we adopt the notion of "potential infinity," we would say that, no, $0.999\ldots$ is not exactly equal to 1, but we can make is as close to 1 as we please by simply adding enough 9's. If we adopt the notion of actual infinity, then we would say, yes, $0.999\ldots$ is exactly equal to 1, because the number exists regardless of our ability to render it in a finite amount of time (like an infinitely long line exists even though it would take us forever to actually draw one). Since the number $0.999\ldots$ exists, and since it is easy to prove it is closer to 1 than any given fixed number, it must be exactly equal to one since there is no number between it and 1.

2. From "Jesuits May Have Helped Newton to 'Discover' Calculus," Manchester University, August 13, 2007.

   (a) According to the author of the article, what little known school in southwest India is responsible for the development foundational Calculus concepts?

   Answer: The Kerala School.

   (b) What particular mathematical discovery was made by this school?

   Answer: They discovered infinite series and, in particular, the Pi series, which they used to calculate Pi correctly to 9, 10 and later 17 decimal places.

   (c) The author lists many reasons to believe this discoveries by this school were brought back to the West via Jesuit missionaries. List two of those reasons.

   Answer:

   - It's hard to imagine that the West would abandon a 500-year-old tradition of importing knowledge and books from India and the Islamic world.
   - "There was plenty of opportunity to collect the information as European Jesuits were present in the area at that time. They were learned with a strong background in maths and were well versed in the local languages."
   - "And there was strong motivation: Pope Gregory XIII set up a committee to look into modernising the Julian calendar. On the committee was the German Jesuit astronomer/mathematician Clavius who repeatedly requested information on how people constructed calendars in other parts of the world. The Kerala School was undoubtedly a leading light in this area."
   - "Similarly there was a rising need for better navigational methods including keeping accurate time on voyages of exploration and large prizes were offered to mathematicians who specialised in astronomy. Again, there were many such requests for information across the world from leading Jesuit researchers in Europe. Kerala mathematicians were hugely skilled in this area."