

# Introduction

1. The Issue - infinity (no intuition)

2. Sizes of Infinity - there are different sizes of  $\infty$  (Georg Cantor)

3. Ex: 
$$\left. \begin{aligned} (1 + -1) + (1 + -1) + (1 + -1) + \dots &= 0 + 0 + 0 + \dots = 0 \\ 1 + (-1 + 1) + (-1 + 1) + (-1 + 1) + \dots &= 1 + 0 + 0 + \dots = 1 \end{aligned} \right\} \text{Grandi's Series}$$

$$T = 1 + -1 + 1 + -1 + 1 + -1 + \dots$$

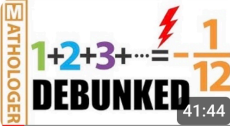
$$+ \quad T = \quad 1 + -1 + 1 + -1 + 1 + -1 + \dots$$

$$2T = 1 + 0 + 0 + \dots$$

$$2T = 1$$

$$T = \frac{1}{2}$$

4. Ex:  $1 + 2 + 3 + 4 + \dots = \cancel{\infty} = -\frac{1}{12}$



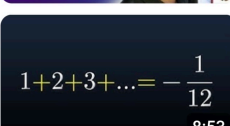
Numberphile v. Math: the truth about  $1+2+3+\dots = -1/12$   
Mathologer  
3M views • 6 years ago



One minus one plus one minus one - Numberphile  
Numberphile  
4.6M views • 11 years ago



Sum of Natural Numbers (second proof and extra...)  
Numberphile  
1.7M views • 9 years ago



This Result Keeps Me Up At Night  
BriTheMathGuy  
987K views • 2 years ago

5. Ex:  $0.999\dots$  exactly equals 1.

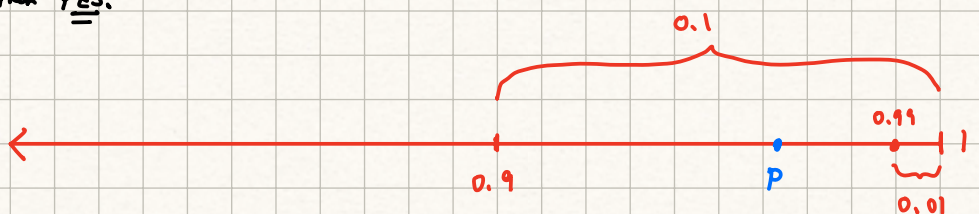
If by exactly equals 1, we mean that at some point, we will have added enough 9's to get to 1, then NO.

If by exactly equals 1, we mean there is no number between  $0.999\dots$  and 1, then YES.

Number vs. Numeral

Number  
↓  
Idea that refers to a

Numeral  
↓  
Symbol we use to represent a number





|                          | 0 | 1 | 2 | 3 |
|--------------------------|---|---|---|---|
| <b>ADLaM</b>             | ⵐ | ⵎ | ⵎ | ⵎ |
| <b>Aegean</b>            |   | ⲁ | Ⲃ | ⲃ |
| <b>Ahom</b>              | ᱀ | ᱁ | ᱂ | ᱃ |
| <b>Arabic</b>            | ٠ | ١ | ٢ | ٣ |
| <b>Balinese</b>          | ᮀ | ᮁ | ᮂ | ᮃ |
| <b>Bamum</b>             |   | ᮊ | ᮋ | ᮌ |
| <b>Bengali</b>           | ০ | ১ | ২ | ৩ |
| <b>Bhaiksuki</b>         | ᱀ | ᱁ | ᱂ | ᱃ |
| <b>Brahmi</b>            |   | — | = | ≡ |
| <b>Chakma</b>            | ᱀ | ᱁ | ᱂ | ᱃ |
| <b>Cham</b>              | ᱀ | ᱁ | ᱂ | ᱃ |
| <b>Chinese (simple)</b>  | 〇 | 一 | 二 | 三 |
| <b>Chinese (complex)</b> | 零 | 壹 | 貳 | 參 |

|   |    |     |    |   |    |     |      |    |   |
|---|----|-----|----|---|----|-----|------|----|---|
|   |    |     |    |   |    |     |      |    |   |
| 0 | 1  | 2   | 3  | 4 | 5  | 6   | 7    | 8  | 9 |
| ○ | ㊀  | ㊁   | ㊂  | ㊃ | ㊄  | ㊅   | ㊆    | ㊇  | ㊈ |
| 一 | 二  | 三   | 四  | 五 | 六  | 七   | 八    | 九  |   |
| 零 | 壹  | 貳   | 參  | 肆 | 伍  | 陸   | 柒    | 捌  | 玖 |
| I | II | III | IV | V | VI | VII | VIII | IX |   |

$$9x = 9$$
$$x = 1$$

7. Ex:

'Alien Calculus' Could Save Particle Physics From Infinities Quanta Magazine.pdf  
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**How to Tame the Endless Infinities Hiding in the Heart of Particle Physics**

*Resurgence*  
Écalle (70 yrs dormant)

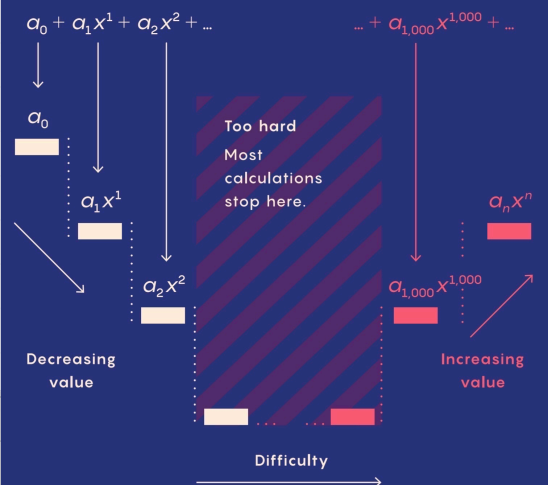
**MATHEMATICAL PHYSICS**

**How to Tame the Endless Infinities Hiding in the Heart of Particle Physics**

🔍 📄 📖

*In the math of particle physics, every calculation should result in infinity. Physicists get around this by just ignoring certain parts of the equations — an approach that provides approximate answers. But by using the techniques known as “resurgence,” researchers hope to end the infinities and end up with perfectly precise predictions.*

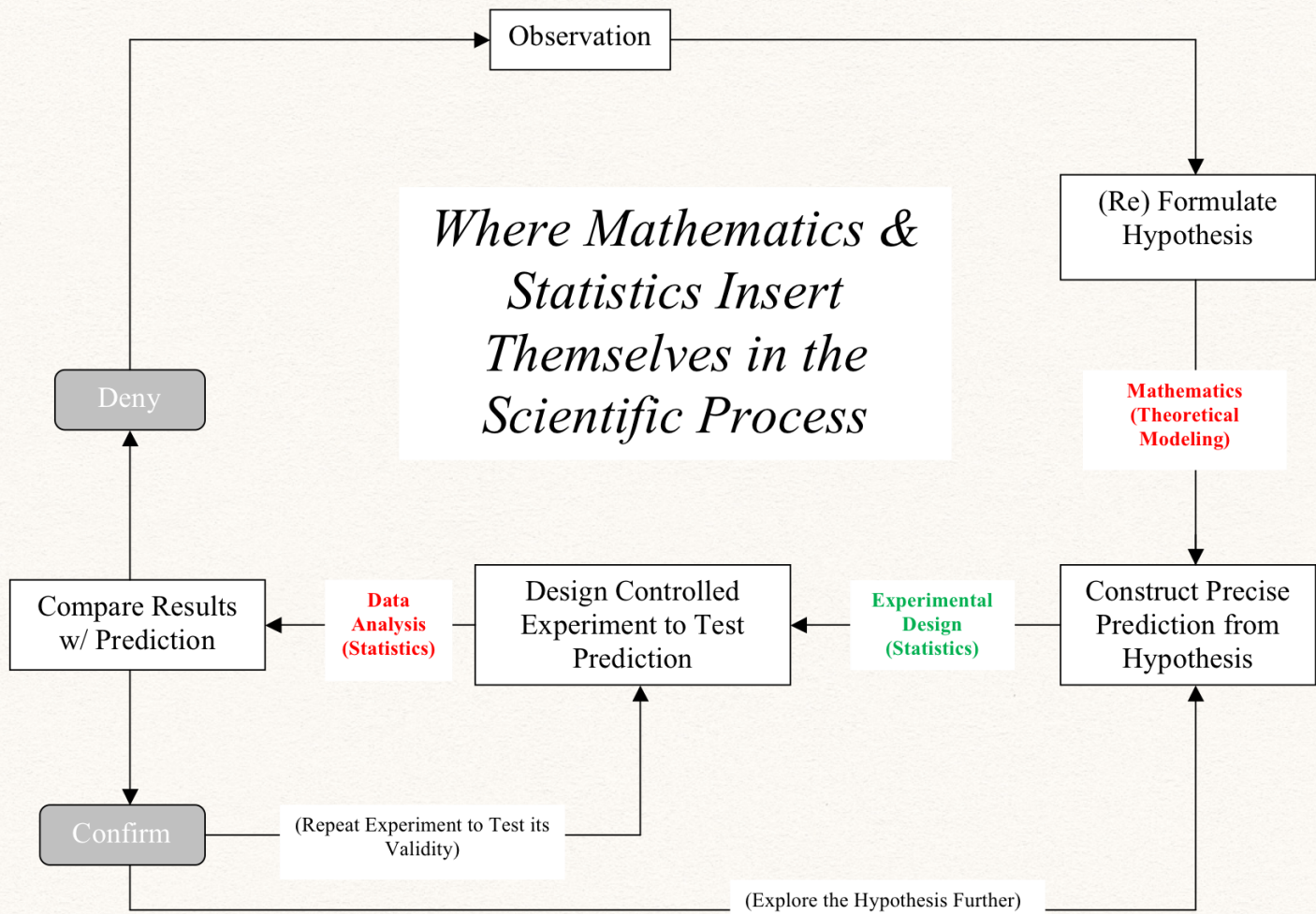
Physicists usually study quantum fields, such as the electromagnetic field, using unending sums. The first few terms in these sums get smaller and smaller, providing an approximate answer. Later terms explode, and the sum seems to lose all meaning.

$$a_0 + a_1x^1 + a_2x^2 + \dots \qquad \dots + a_{1,000}x^{1,000} + \dots$$




# Understanding Modeling

## 1. Overview of the Scientific Method -



## 2. Mathematical Model - expression (equation or function) which quantitatively represents a phenomenon.

Ideally, it should be both descriptive and predictive.

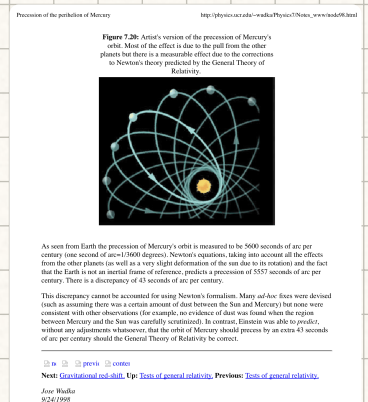
- Descriptive - accurately represents a known and measurable phenomenon.
- Predictive - accurately represents a measurable but previously unknown phenomenon.

### Ex: Einstein's General Theory of Relativity

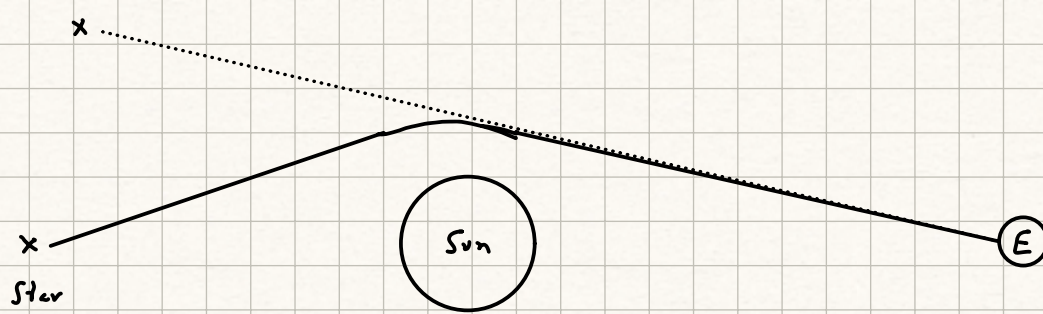
- Newton - gravity is a force; derive shape of orbits of planets to be ellipses.
- Einstein - gravity is an effect of warped space.

① Descriptive - it accurately described the precession in planetary orbits.

② Predictive - anything that travels through space, including light, should be affected by gravity.







Ex: Mendeleev (1869) - Periodic Table

- Description - organised elements based on shared properties.
- Predictive - predicted the existence of previously unknown elements.

3. Statistical - uncovers useful patterns in data, and it can sometimes have predictive value, but it gives no insight into the underlying mechanisms causing a behavior.

Ex: Mendeleev + Periodic Chart - predicted existence of unknown elements but gives no insight into why elements are periodic or arranged in the pattern.

Ex: Actuarial Sciences

M/F driver's insurance rates

- No insight into why M are higher risk.
- Why?
  - Testosterone
  - Socialization
  - Frontal Lobe Development
 } underlying mechanisms

4. Theoretical Models - built from first principles (fundamental observation of physical law of universe) and they give insight into the underlying mechanisms causing a behavior.

- Proportional - two quantities are proportional iff one is a constant multiple of the other.
- Inversely Proportional - ... iff one is proportional to the reciprocal of the other.
- Jointly Proportional - ... iff one is proportional to the product of the others.



Ex:  $P$  = pressure of gas in a closed container

$T$  = temp. of the gas

$$P \propto T$$

$$P = kT$$

$k = \frac{P}{T}$  } should be constant for different values of  $T, P$ .

---

$n$  = amount of gas in container (no. of moles)

$$P \propto n$$

$$P = kn$$

---

$V$  = volume of the container

$$P \propto \frac{1}{V}$$

$$P = k \cdot \frac{1}{V}$$

$\therefore P$  is jointly proportional to  $n$  and  $T$  and inversely proportional to  $V$ .

$$P \propto n \cdot T \cdot \frac{1}{V}$$

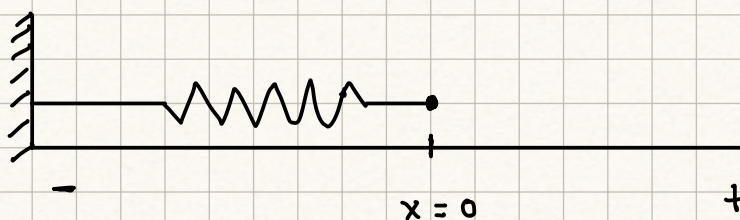
$$P = \textcolor{red}{R} \cdot n \cdot T \cdot \frac{1}{V}$$

↑  
prop. constant

$$PV = nRT \quad (\text{Ideal Gas Law})$$

Ex:

Hooke's Law says that the force a spring exerts is proportional to its distortion.



$$F \propto x$$

$$F = \textcolor{red}{k} \cdot x$$

↑



proportionality constant

What factors affect the value of  $k$ ?

[List everything that might affect  $F$ ]

- $x$  {
- distortion
- $k$  {
- type of material
  - temperature
  - tightness of the coil
  - size of the material that coiled
  - length
  - other stuff we haven't thought of...

$k$  is constant with respect to the other variables in the equation:  $F, x$

List of factors affecting  $k$  tell us when the model is valid.

How do we find the value of  $k$ ?

$$F = kx, \text{ so } k = \frac{F}{x}$$

| $x$ | $F$  | $k = \frac{F}{x}$ |
|-----|------|-------------------|
| 1.2 | 3.8  | 3.17              |
| 3.6 | 13.4 | 3.72              |
| 4.8 | 17.9 | 3.73              |
| ⋮   | ⋮    | ⋮                 |
| ⋮   | ⋮    | ⋮                 |
| ⋮   | ⋮    | ⋮                 |

} roughly constant;  
 $\bar{k} = \frac{3.17 + 3.72 + 3.73}{3}$   
 $= 3.54$

Ex: Epidemiology - risk  $R$  of contracting an airborne illness is jointly proportional to the concentration  $C$  of the pathogen in the air and the time of exposure,  $T$ .

$$R \propto C \cdot T$$

$$R = k C \cdot T$$

↑  
List factors affecting  $k$

[List ...  $R$ ]



CT {

- concentration
- time of exposure

K {

- age
- strength of your immune system
- air currents / filtering
- contagiousness of the pathogen
- size of room
- immunization
- natural immunity
- sleep
- diet + lifestyle

Ex:

Allometry

$$SA \propto M?$$

$$SA = k \cdot M?$$

Relationship between M and V

$$\text{density, } \rho = \frac{M}{V} \rightarrow V = \frac{1}{\rho} \cdot M$$

Relationship between SA and V

$$cm^2 = (cm^3)^{\frac{2}{3}}$$

$$SA \propto V^{\frac{2}{3}}$$

} dimensional analysis

$$\therefore SA \propto V^{\frac{2}{3}} \propto M^{\frac{2}{3}}$$

$$SA \propto M^{\frac{2}{3}}$$

$$SA = k \cdot M^{\frac{2}{3}}$$

↑ what affects this value?

[List all factors that might affect SA.]

- ~~M~~
- shape
- species
- age
- height



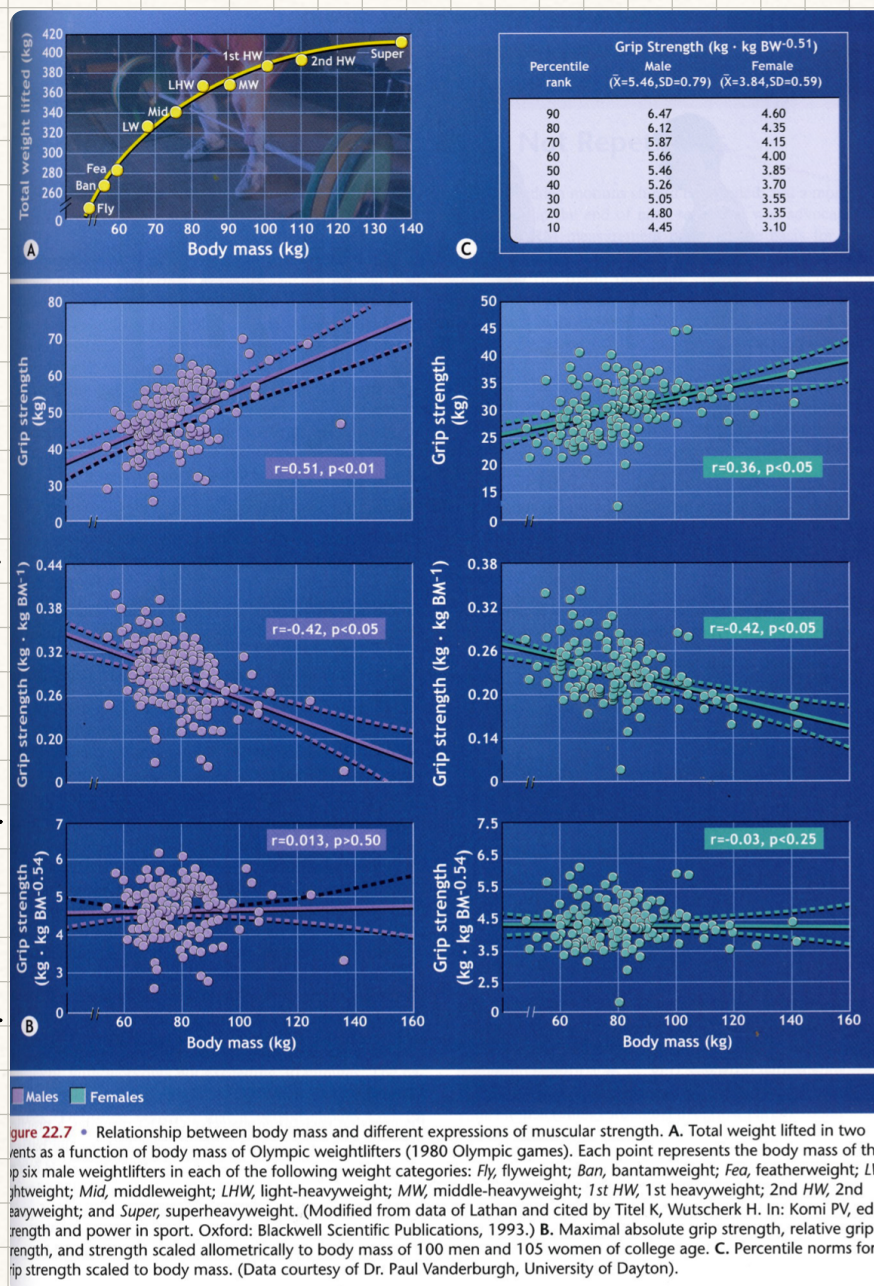
- $k$  {
- temperature
  - pregnancy
  - nutrition / diet
  - condition (missing limbs)
  - sex

| M   | SA               | $k = \frac{SA}{M^{2/3}}$ |
|-----|------------------|--------------------------|
| 2.1 | 1.20427770541849 | 0.734368065913272        |
| 5.2 | 2.18326648960056 | 0.727396465643218        |
| 3.7 | 1.74520813842199 | 0.72953491239041         |
| 4.8 | 2.07112685874293 | 0.727856686103162        |

0.729

Average is  $\frac{0.729}{4} = k$

Ex: Grip Strength vs. Body Mass



$$GS \cdot BM^{-1}$$

$$= \frac{GS}{BM}$$

BM out-strips  
GS as BM  
grows

$$GS \cdot BM^{-0.54}$$

$$= \frac{GS}{BM^{0.54}}$$

$y = 4.6$  is equation of the horizontal line

$$\frac{GS}{BM^{0.54}} = 4.6 \longrightarrow GS = 4.6 \cdot BM^{0.54} \longrightarrow GS \propto BM^{0.54}, \text{ maybe } GS \propto \sqrt{BM}$$



## Linear Functions and Rate of Change

1. Linear Functions -  $y$  is a linear function of  $x$  iff  $y$  increases by a constant amount for each unit inc. in  $x$ .  
(The constant amount is called slope.)

$$y = mx + b$$

↑                      ↑  
slope                  initial value of  $y$   
                                 ( $y$ -intercept)

$m$  and  $b$  are parameters (constants).

### Assumed Skills -

1. Find the equation of a line given 2 points:  $(x,y) = (1,3)$  and  $(7,-2)$ .
2. Find equation of a line given slope and a point: slope is 2, through  $(7,4)$ .
3. Find equation of a line given its initial value and rate of inc./dec.: initial value is 3.4 and each unit inc. in  $x$  gives a decrease of 4.8 units in  $y$ .

Ex: For #1.

$$m = \frac{-2 - 3}{7 - 1} = -\frac{5}{6}$$

$$y = -\frac{5}{6}x + b$$

$$3 = -\frac{5}{6} \cdot 1 + b$$

$$3 + \frac{5}{6} = b$$

$$\frac{23}{6} = b$$

$$y = -\frac{5}{6}x + \frac{23}{6}$$

Ex: For #3.

$$y = -4.8x + 3.4$$

$$-\frac{4.8}{1}$$



Ex:

T-Shirts

$$C = 16.37g + 38$$

cost of production for  $g$  shirts

no. of shirts produced

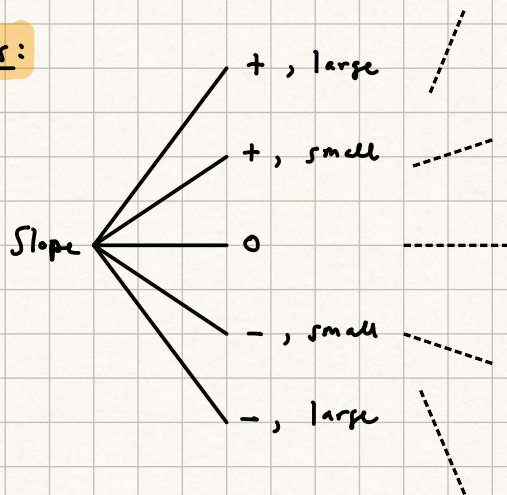
variable costs

fixed costs (unavoidable)

What is the cost of production per shirt?

$$\begin{aligned}\frac{C}{g} &= \frac{16.37g + 38}{g} = \frac{16.37g}{g} + \frac{38}{g} \\ &= 16.37 + \frac{38}{g}\end{aligned}$$

Notes:



## 2. Rate of Change -

Ex:

Deborah Number (example of fractions being used to compare 2 quantities)

$$D = \frac{T_R}{T_0} = \frac{\text{Time of Relaxation}}{\text{Time of Observation}} = \begin{cases} \text{Large if } T_R \gg T_0 & (\text{Solid}) \\ \text{Small if } T_R \ll T_0 & (\text{Liquid}) \end{cases}$$

Ex:

$$C = 6.37g + 38$$

$C$  is not proportional to  $g$ .

However,  $\Delta C$  is proportional to  $\Delta g$

$$\Delta C = 6.37 \cdot \Delta g$$

$$\frac{\Delta C}{\Delta g} = 6.37 \text{ (slope)}$$



This means that any increase in  $g$  will correspond to an increase in  $C$  that is 6.37 times as large as the increase in  $g$ .

Rate of Change of  $C$  with respect to  $g = \frac{\Delta C}{\Delta g} = 6.37$

6.37 is a multiplier (sensitivity).

$\Delta g$  is an error in the measurement of  $g$ .

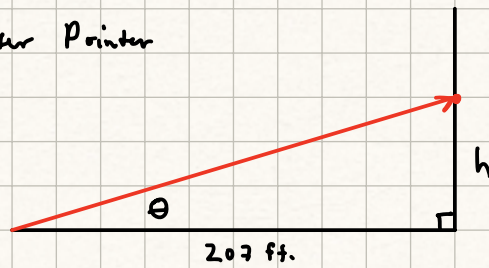
$\Delta g$  is variability in  $g$ .

For example, if we measure  $g = 2.7 \pm 0.03$  cm, then the induced error in our computed

value of  $C$  would be  $\Delta C = 6.37 \cdot \Delta g = 6.37 (\pm 0.03) = \pm 0.191$  cm.

Ex:

Laser Pointer



$$\frac{\Delta h}{\Delta \theta} = 29.2 \frac{\text{ft}}{\text{deg}}$$

$$\Delta h = 29.2 \cdot \Delta \theta$$

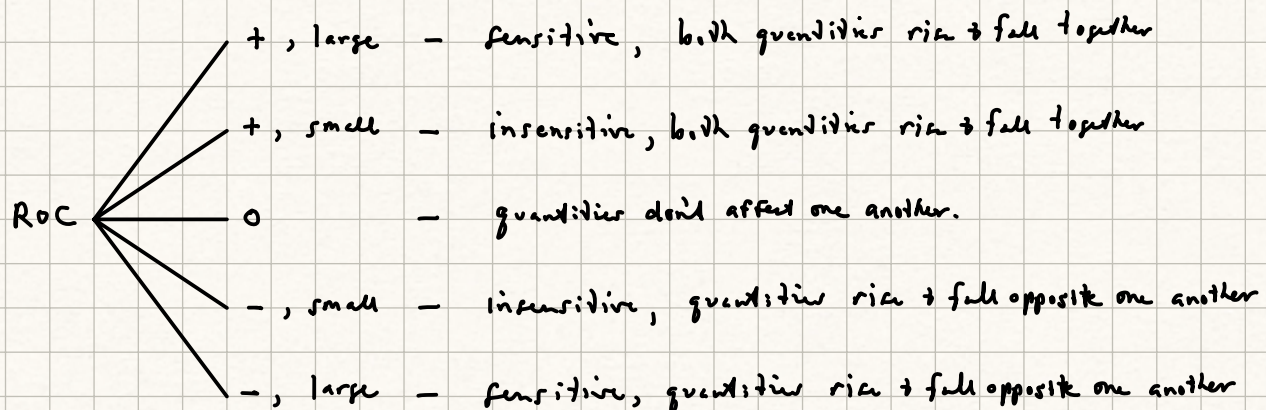
↑ multiplier  
↑ variability in  $\theta$  (hand shaking)

(Error enhancing situation)

Ex:

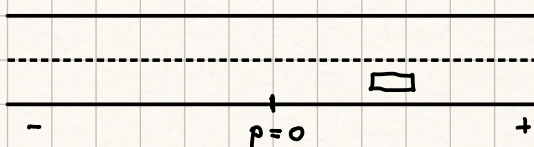
Movie Sensitivity

Note:



Ex:

Let  $p$  = position of a car on a road, in ft., at any given time  $t$ , in seconds.





$$\frac{\Delta p}{\Delta t} = 8.7 \frac{\text{ft}}{\text{s}} \quad (\text{speed, velocity})$$

$$\Delta p = 8.7 \frac{\text{ft}}{\text{s}} \cdot \Delta t = 8.7 \cdot \Delta t \text{ ft}$$

Ex: Let  $p$  = pressure (in atm) at a depth of  $d$  meters below water.

- ① If  $p(5) = 1.8$ , give the units on the numbers and explain the meaning of the statement.
- $\uparrow$                        $\uparrow$   
 m                      atm

When depth is 5 m, the pressure is 1.8 atm.

- ② Do we expect the ROC to be + or - ? Explain.

Positive, since a depth inc., pressure inc. as well.

- ③ Suppose the ROC of  $p$  wrt  $d$  is 1.2. Give the units on the number & explain the meaning of the statement.
- $\swarrow$  <sup>output</sup>                       $\swarrow$  <sup>input</sup>  
 $p$                        $d$

$$\frac{\Delta \text{out}}{\Delta \text{in}} = \frac{\Delta p}{\Delta d} = \left( \frac{\text{atm}}{\text{m}} \right)$$

$$\frac{\Delta p}{\Delta d} = 1.2$$

$$\Delta p = 1.2 \cdot \Delta d$$

any inc. in depth corresponds to an inc. in pressure that is 1.2 times as large as the inc. in depth.

- ④ If  $p(3) = 1.7$  and the ROC of  $p$  wrt  $d$  is 0.6, estimate the values of  $p(3.5)$  and  $p(2.8)$ .

$$\frac{\Delta p}{\Delta d} = 0.6 \longrightarrow \Delta p = 0.6 \cdot \Delta d$$

•  $p(3.5)$  :

$$\Delta d = 3.5 - 3 = 0.5 \text{ m}$$

$$\Delta p = 0.6 \cdot \Delta d = 0.6 \cdot 0.5 = 0.3 \text{ atm}$$

$$p(3.5) = p(3) + 0.3 = 1.7 + 0.3 = \boxed{2.0 \text{ atm}}$$



•  $p(2.8)$ :

$$\Delta d = 2.8 - 3 = -0.2 \text{ m}$$

$$\Delta p = 0.6 \cdot \Delta d = (0.6)(-0.2) = -0.12 \text{ atm}$$

$$p(2.8) = p(3) + -0.12 = 1.7 - 0.12 = \boxed{1.58 \text{ atm}}$$

⑤ If  $p(3) = 1.7$  and ROC of  $p$  wrt  $d$  is  $0.6 \frac{\text{atm}}{\text{m}}$ ,

• If we measure  $d = 3 \pm 0.4 \text{ m}$ , find the induced error in our calculation of  $p$ .

$$\begin{aligned} \frac{\Delta p}{\Delta d} &= 0.6 \longrightarrow \Delta p = 0.6 \cdot \Delta d \\ &= 0.6 \cdot (\pm 0.4) \\ &= \pm 0.24 \text{ atm} \end{aligned}$$

• If we wish to maintain a pressure of  $1.7 \pm 0.2 \text{ atm}$  on our object, how tightly must we control the depth of our object.

$$\begin{aligned} \Delta p &= 0.6 \cdot \Delta d \\ \pm 0.2 &= 0.6 \cdot \Delta d \\ \frac{\pm 0.2}{0.6} &= \Delta d \\ \boxed{\pm 0.33 \text{ m} = \Delta d} \end{aligned}$$

Ex:

$C$  = cost of producing  $g$  items

$R$  = revenue made from selling  $g$  items

ROC of  $C$  with respect to  $g$  is  $10.8$ :

$$\frac{\Delta C}{\Delta g} = 10.8 \frac{\$}{\text{item}}$$

$$\Delta C = 10.8 \cdot \Delta g$$



Any unit inc. in  $g$  will result in an inc. in  $C$  that is  $10.8$  times larger. [The cost of producing one more unit is \$10.80.]



ROC of R with respect to g is 9.7:

$$\frac{\Delta R}{\Delta g} = 9.7 \frac{\$}{\text{item}}$$

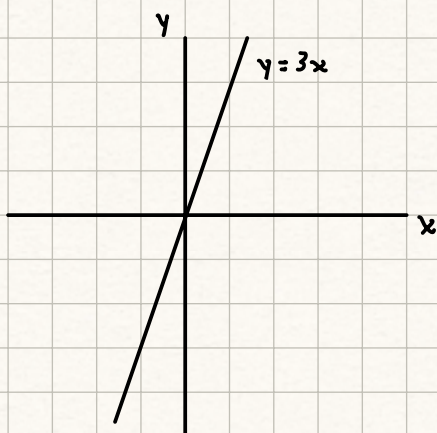
$$\Delta R = 9.7 \cdot \Delta g$$

Any unit inc in g will result in an inc. in R that is 9.7 times as large. [The amount of revenue gained from selling one more item is \$9.70.]

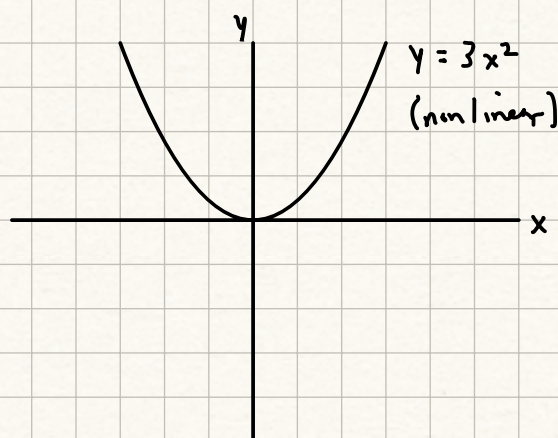
### Nonlinear Functions

1. Idea - for linear functions, the ROC is constant (which means the change in output is proportional to the change in input). What if this is not true? What if the proportionality "constant" is not constant?

2. Ex:  $y = 3x$  ROC is slope



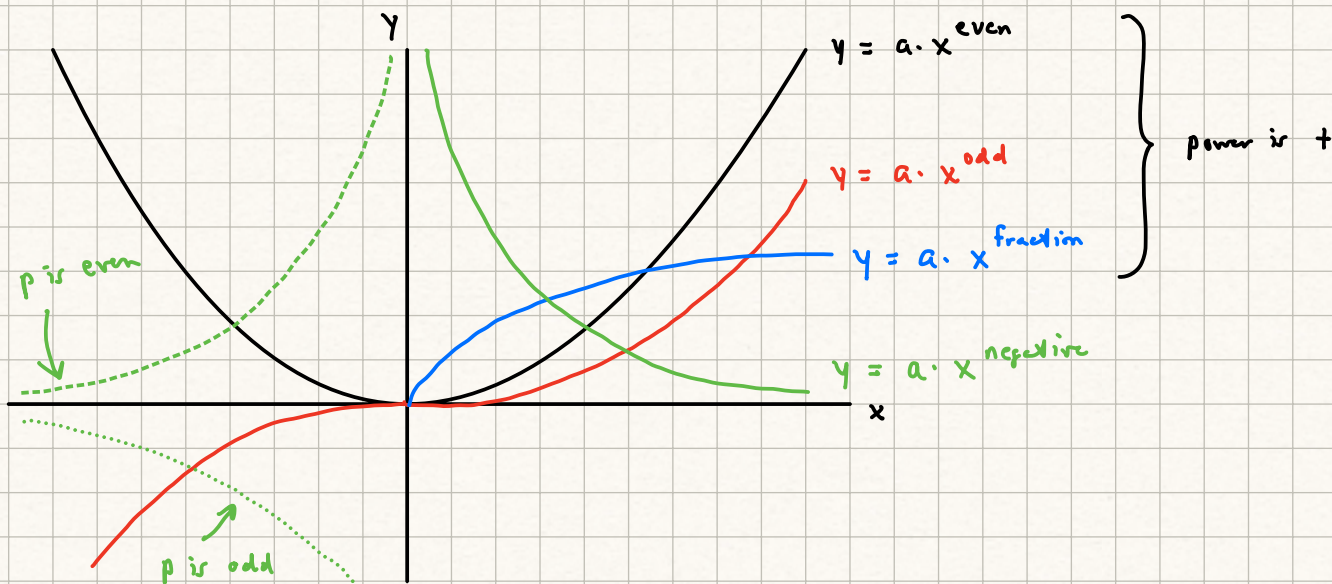
$y = 3x^2$  ROC is no longer constant



3. Power Functions -

$y = a \cdot x^p$

- $a$ : affects how fast function grows
- $x$ : variable is in the base
- $p$ :
  - constant
  - + growth
  - decay





## Recall:

- ① Negative exponents changes numerator and denominator.

Ex:  $y = x^{-2} = \frac{1}{x^2}$

$$y = \frac{1}{x^{-3}} = x^3$$

- ② Fractional exponents translate to roots and powers:

$x^{\frac{p}{r}}$   
p ← power  
r ← root

Ex:  $y = x^{\frac{3}{4}} = \sqrt[4]{x^3}$  or  $(\sqrt[4]{x})^3$

$$y = \sqrt[5]{x^3} = x^{\frac{3}{5}}$$

$$y = \frac{6}{x^{-3/2}} = 6x^{3/2} = 6\sqrt{x^3}$$

## 4. Exponential Functions-

$y$  is an exponential function of  $x$  iff  $y$  inc. or dec. by a constant percent for each unit inc. in  $x$ .

$$y = a \cdot b^x$$

constants (parameters)

Ex:

$$y = 78(1.38)^x$$

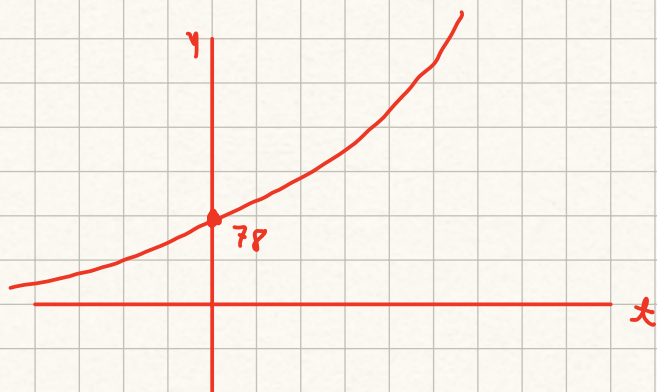
38% growth  
initial output ( $y$ -intercept)

$$[\text{If } x=1, y = 78(1.38)^1 = 78(1 + 0.38)]$$

$$= 78 + 78 \cdot 0.38$$

↑  
Additional 38%

- Identify if it is growth or decay.
- Explain why.  $1.38 > 1$
- Identify the initial value. 78
- Identify growth/decay rate. 38%
- Sketch graph.





Ex:

$$y = 2.3 (0.81)^x$$

• Identify if it is growth or decay.

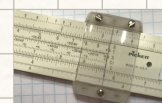
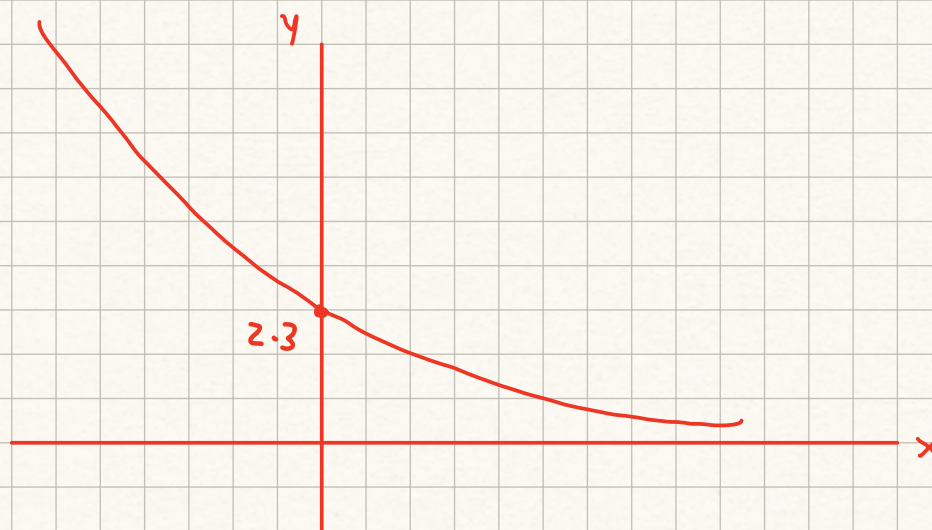
• Explain why.  $0.81 < 1$

• Identify the initial value.  $2.3$

• Identify growth/decay rate.  $19\%$

$$1 - 0.81 = 0.19 = 19\%$$

• Sketch graph.



Ex:

$$y = -2.5 (1.72)^x$$

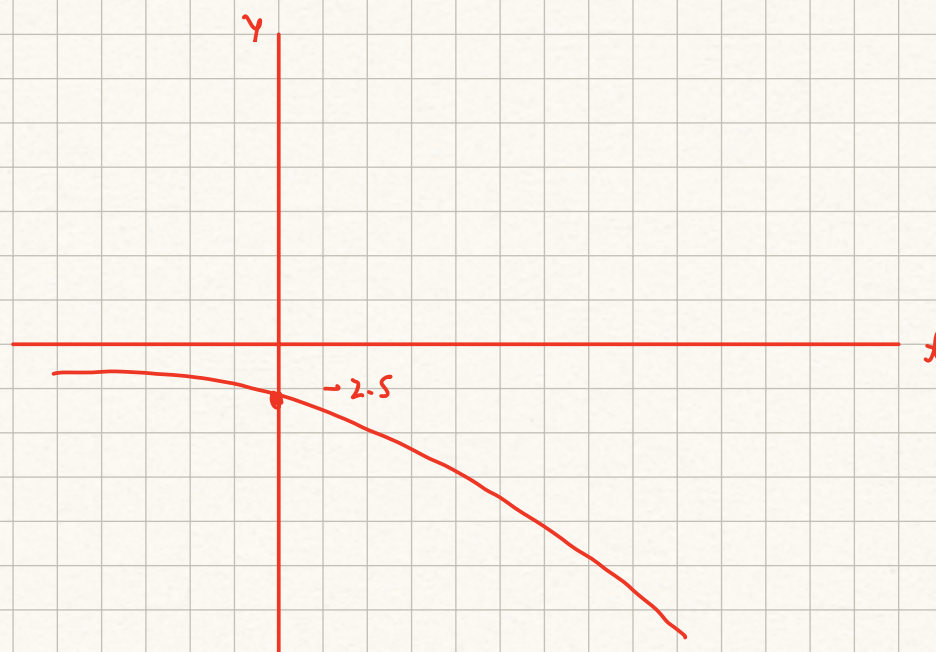
• Identify if it is growth or decay.

• Explain why.  $1.72 > 1$

• Identify the initial value.  $-2.5$

• Identify growth/decay rate.  $72\%$

• Sketch graph.





Increasing — as a graph is read from left to right, it gets higher.

Growth — as a graph is read from left to right, it gets further from horizontal axis.

Decreasing vs. Decay are similarly defined.

Ex:

$$y = -2.5 (3.72)^x$$

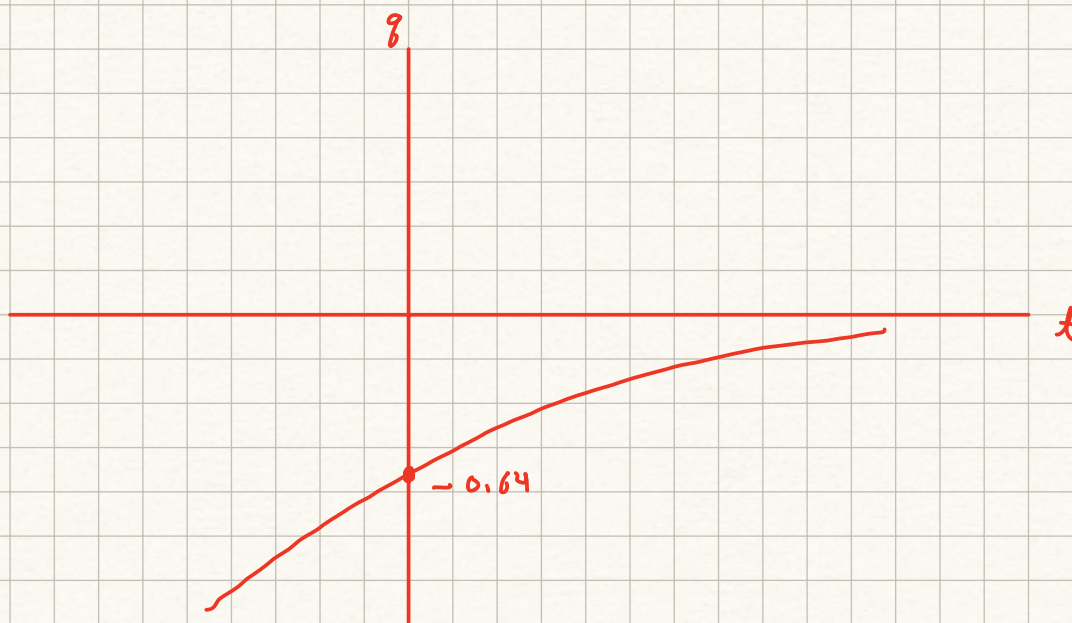
What is the percent growth rate?

$$3.72 - 1 = 2.72 = 272\%$$

Ex:

$$g = -0.64 (0.12)^x$$

- Identify if it is growth or decay.
- Explain why.  $0.12 < 1$
- Identify the initial value.  $-0.64$
- Identify growth/decay rate.  $88\%$   $1 - 0.12 = 0.88 = 88\%$
- Sketch graph.



Ex:

$$y = 3.81 (1.2)^{3x}$$

$$= 3.81 [(1.2)^3]^x$$

$$= 3.81 [1.728]^x$$

Growth Rate is  $72.8\%$ , not  $20\%$



$$y = 3.81 (0.92)^{4x}$$

$$= 3.81 (0.92^4)^x$$

$$= 3.81 (0.716)^x$$

↖ 28.4% since  $1 - 0.716 = 0.284$

$$y = 3.81 (0.92)^{-x}$$

$$= 3.81 (0.92^{-1})^x$$

$$= 3.81 (1.087)^x$$

↖ 8.7% growth

Ex:

Write an exponential function with a 72% growth rate in the form

$$y = 3.81 (0.92)^{rx}$$

$$= 3.81 (0.92^r)^x$$

↖ want this to turn out to be 1.72

$$0.92^r = 1.72$$

$$\log(0.92^r) = \log(1.72)$$

$$r \cdot \log(0.92) = \log(1.72)$$

$$r = \frac{\log(1.72)}{\log(0.92)} = \frac{0.236}{-0.036} = -6.6$$

$$\therefore y = 3.81 (0.92)^{-6.6x}$$

Note:

We can write any exponential growth or decay amount using any base we want.

Most common bases are 10 and  $e = 2.71828...$

$$y = a \cdot 10^{rx} \quad \text{or} \quad y = a e^{rx}$$

Ex:

Write  $y = 5.7 e^{0.2x}$  in the form  $y = a \cdot b^x$

$$y = 5.7 (e^{0.2})^x$$

$$= 5.7 (1.221)^x$$

↖ 22.1% growth rate (per unit time)



Ex:

Write  $y = 7.2 e^{-0.34t}$  in the form  $y = a \cdot b^t$  and identify both continuous and non-continuous growth/decay rate.

Decay since  $-0.34 < 0$  (negative)

34% continuous rate

$$y = 7.2 (e^{-0.34})^t$$

$$= 7.2 (0.712)^t$$

$\uparrow$   $1 - 0.712 = 0.288 = 28.8\%$  non-continuous rate

Ex:

Find the continuous and non-continuous growth rate for

$$y = 7.9 (1.13)^t$$

13% non-continuous rate

$$y = 7.9 e^{rt} = 7.9 (e^r)^t = 7.9 e^{0.122t}$$

12.2% continuous rate

Need  $e^r = 1.13 \longrightarrow e^r = 1.13$

~~$$\log(e^r) = \log(1.13)$$~~

$$r = \ln(1.13) = 0.122$$

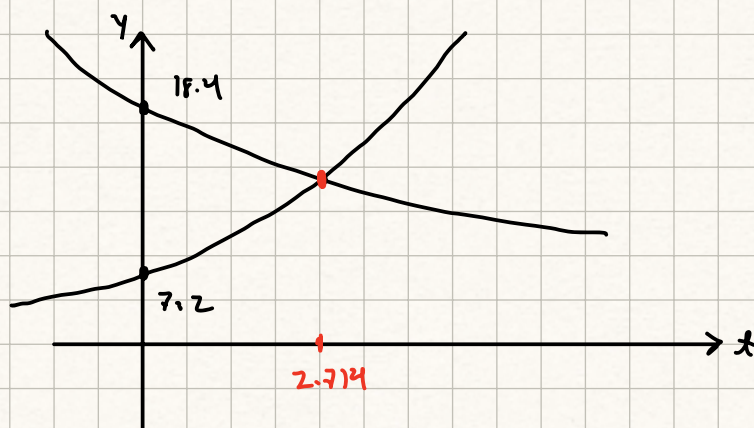
~~$$r \cdot \log(e) = \log(1.13)$$~~

~~$$r = \frac{\log(1.13)}{\log(e)} = \frac{0.053}{0.434} = 0.122$$~~

Ex:

Do  $y = 7.2 (1.3)^t$  and  $y = 18.4 (0.92)^t$  intersect for a positive  $t$  value?

If so, find the intersection.



$$7.2 (1.3)^t = 18.4 (0.92)^t$$

$$\frac{(1.3)^t}{(0.92)^t} = \frac{18.4}{7.2}$$



$$\left(\frac{1.3}{0.92}\right)^x = \frac{18.4}{7.2}$$

$$1.413^x = 2.556$$

$$\ln(1.413^x) = \ln(2.556)$$

$$x \cdot \ln(1.413) = \ln(2.556)$$

$$x = \frac{\ln(2.556)}{\ln(1.413)} = 2.714$$

$$x = 2.714$$

Ex:

Solve for  $x$ :  $7.2 e^{1.3x} = 18.4 e^{-0.93x}$

$$\frac{e^{1.3x}}{e^{-0.93x}} = \frac{18.4}{7.2}$$

$$e^{2.23x} = 2.556$$

$$\ln(e^{2.23x}) = \ln(2.556)$$

$$2.23x = 0.938$$

$$x = 0.421$$

## 5. Logarithms - track exponents

$$\log 10^7 = 7$$

$$\log 24 = 1.38 \rightarrow 10^{1.38} = 24$$

$$\log 1 = 0 \rightarrow 10^0 = 1$$

$$\log 0.01 = -2 \rightarrow 10^{-2}$$

$$\log(-3) = ? \rightarrow 10^? = -3$$

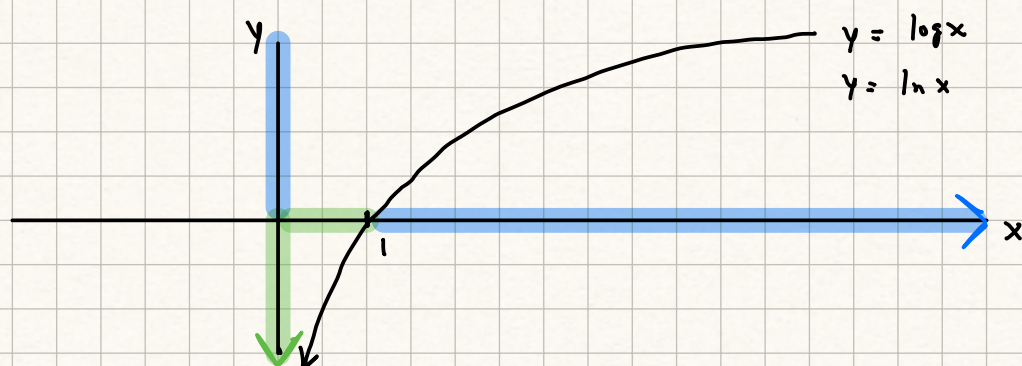
↑  
DNE

$$\ln 10^7 = \log_e 10^7 = 16.12 \rightarrow e^{16.12} = 10^7$$

.

.

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Logs help us process very large and very small numbers:

- Scientific Notation (e.g.  $10^{77}$ )
- OxyContin Scandal

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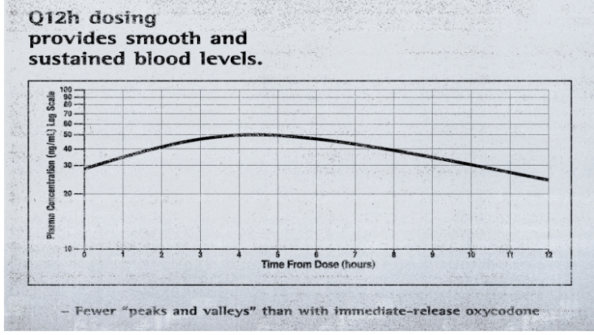
## How Purdue Used Misleading Charts to Hide OxyContin's Addictive Power

MONEY WATCH BY JIM EDWARDS UPDATED ON: SEPTEMBER 28, 2011 / 1:36 PM / MONEYWATCH f t v

A case filed by the Commonwealth of Kentucky against Purdue Pharma sheds new light on the origin of an infamously misleading chart showing that OxyContin stays in patients' blood levels over time, when in fact levels of the drug drop off sharply in users, triggering withdrawal symptoms and addiction. The chart was later used by Purdue as a marketing tool to convince doctors that OxyContin was less addictive than it actually was, the case claims.

The suit is unusual because Kentucky isn't just seeking reimbursement for Medicaid payments it made based on misleading marketing for the drug. It's seeking all costs it incurred associated with OxyContin abuse, including millions spent in drug abuse education and prevention, and the "investigation, apprehension, prosecution and incarceration of individuals, who, due to the fraudulently concealed addictive nature of OxyContin, have resorted to criminal means to continue their addiction." Forty people in Pike County, Ky., have died from OxyContin abuse, the case claims. Damages could potentially reach into the billions of dollars.

A version of the chart surfaced in a petition demanding the FDA remove the chart from OxyContin's official labeling. The chart appears to show that OxyContin doses peak quickly and then plateau in the blood stream, providing constant levels of pain relief:

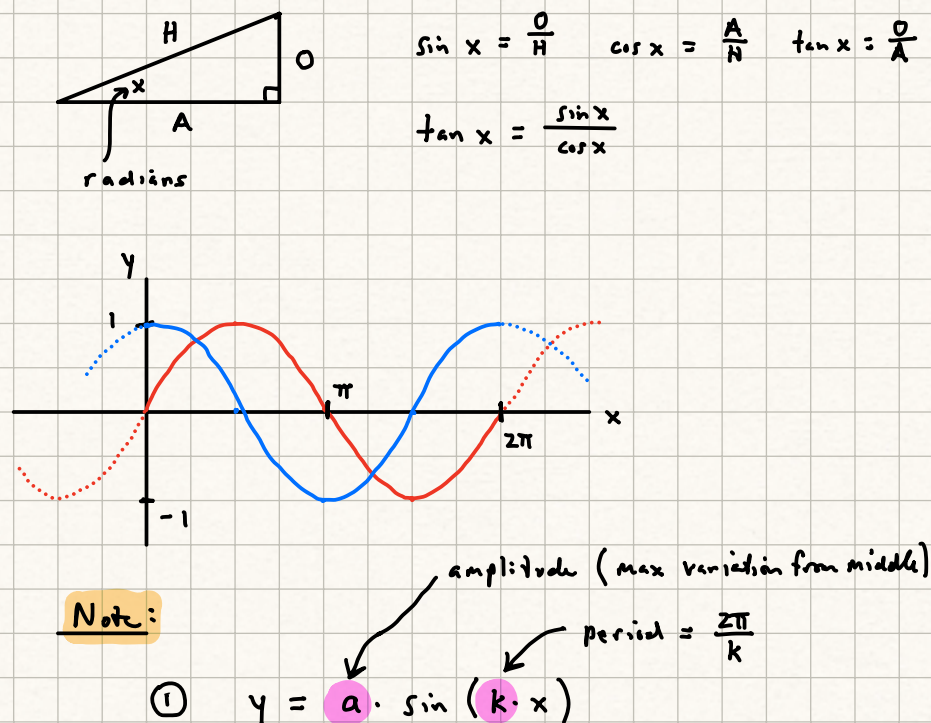


But the chart is in *logarithmic* scale, not *linear* scale. In a log scale, each point on the vertical axis is a multiple of the point below it. On a linear scale -- the scale most people are accustomed to seeing in charts -- the points represent equal intervals. The same data on a linear scale looks like this, showing that OxyContin doses peak quickly and then fall off after a few hours:

The Kentucky case claims that Purdue first developed the chart in a study in the mid 1990s. The chart compared a

- Richter Scale
- pH
- Decibels
- Statistics

## 6. Periodic Functions - repeating





- ② All periodic functions are just the sum of sines and cosines of different amplitudes and periods. (Fourier)