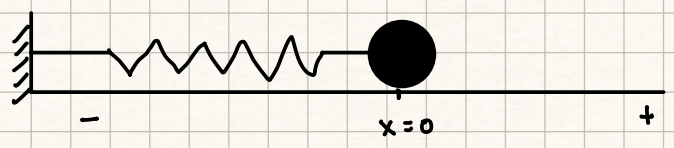
Introduction to Modeling

1. **Model** – a mathematical expression of a phenomenon, which is built on underlying principles.

Ex:



Hooke's Law says that the force exerted by a spring is proportional to its distortion.

$F \propto x$

$F = kx$

k: Proportionality Constant (Parameter)

What affects the value of k?
[List all factors affecting F]

- x
  - Distortion
- k
  - Material
  - Temperature
  - Coiling
  - Thickness of Coiling Material
  - Other stuff?

x cannot affect k if the model is to remain valid.

The model ceases to be valid if any of the factors that affect k change.

How do we find the value of k? Empirically.

$F = kx$

$k = \frac{F}{x}$

x	F	$k = F/x$
2	6	3
0.5	1.5	3
3	9	3

constant

$\therefore k = 3$

Ex: Force of air resistance on an object is proportional to the velocity of the object relative to the air.

$F \propto v$

$F = kv$

k: Proportionality Constant (Parameter)

What affects this value?
[List all factors affecting F]

v { • v
k { • cross-sectional area
• spin
• density of air
• texture
• altitude
• humidity
• orientation

v	F	$k = \frac{F}{v}$
2	8	4
3	18	6
1	2	2
4	32	8

} not constant

∴ model does not fit the data.

k is not constant; it depends on v.

In fact, $k = 2v$

∴ $F = kv = (2v)v$

$F = 2v^2$

$F \propto v^2$

Ex: Newton's Laws of Motion: $F = ma$ (2^{nd})

Newton's Law of Gravity: the force of gravity between 2 objects is <u>jointly proportional</u> to their mass, m and M, and <u>inversely proportional</u> to the square of the distance between their centers of mass.

$F \propto m \cdot M \cdot \frac{1}{d^2}$

$F = G \cdot m \cdot M \cdot \frac{1}{d^2}$

$F = \frac{GmM}{d^2}$

$F = ma$

$a = \frac{GM}{d^2} = 32 \frac{ft}{s^2}$ or $9.81 \frac{m}{s^2}$