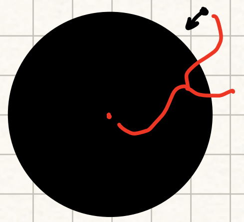
Ex: Newton's Laws of Motion: $F = ma$ (2nd)

Newton's Law of Gravity: the force of gravity between 2 objects is jointly proportional to their mass, m and M, and inversely proportional to the square of the distance between their centers of mass.

$$F \propto m \cdot M \cdot \frac{1}{d^2}$$

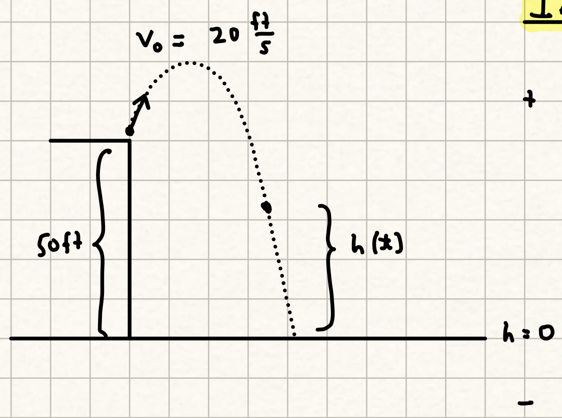


$$F = G \cdot m \cdot M \cdot \frac{1}{d^2}$$

$$\left.\begin{aligned} F &= \frac{GmM}{d^2} \\ F &= ma \end{aligned}\right\} \quad a = \frac{GM}{d^2} = 32\,\frac{ft}{s^2} \text{ or } 9.81\,\frac{m}{s^2}$$

Introduction to Dynamical Systems

1. **Ex:**



$$a = -32 \ \frac{ft}{s^2}$$

$$v' = -32$$

$$\left.\begin{aligned} v &= -32t + C \\ v(0) &= 20 \end{aligned}\right\} \quad -32(0) + C = 20 \longrightarrow C = 20$$

$$v = -32t + 20$$

$$h' = -32t + 20$$

$$\left.\begin{aligned} h &= -32 \cdot \tfrac{1}{2}t^2 + 20t + D \\ h(0) &= 50 \end{aligned}\right\} \quad -16(0)^2 + 20(0) + D = 50 \longrightarrow D = 50$$

$$h = -16t^2 + 20t + 50$$

$$\left.\begin{aligned} &\text{Differential Eqn.} && \{\, h'' = -32 \\ &\text{Initial Conditions} && \left\{\begin{aligned} h'(0) &= 20 \\ h(0) &= 50 \end{aligned}\right. \end{aligned}\right\} \quad \text{Continuous Dynamical System (CDS)}$$

2. **Dynamical System** – any system (model, equation) that depends on time, either implicitly or explicitly.

- **dynamic** – depends on time.
- **static** – time independent (e.g. $PV = nRT$)
- **explicit** – dependence on time is directly indicated in the equation (e.g. $h = -16t^2 + 20t + 50$)
- **implicit** – exact nature of the dependence on time is hidden – implied ($h'' = -32$)
- **continuous** – iff time can be any real number. [CDS]
- **discrete** – iff time can take on only integer values. [DDS]