2. **Dynamical System** – any system (model, equation) that depends on time, either implicitly or explicitly.

- **dynamic** – depends on time.
- **static** – time independent (e.g. $PV = nRT$)
- **explicit** – dependence on time is directly indicated in the equation (e.g. $h = -16t^2 + 20t + 50$)
- **implicit** – exact nature of the dependence on time is hidden – implied ($h'' = -32$)
- **continuous** – iff time can be any real number. [CDS]
- **discrete** – iff time can take on only integer values. [DDS]

3. **Ex:** Malthusian Model of Population Growth

ROC of a population $P \propto P$

$$\frac{dP}{dt} = (k)P$$

(k: constant of proportionality)

What affects this value?

~~Population~~
- Cost of Living
- Disease
- Fertility
- Food
- Wars
- Predators
- Climate
- M/F Ratio
- Species
- Life Expectancy
- Geography
- Space
- Natural Disasters
- Immigration
- Emmigration
- Other

Suppose $k = 3$.

$\frac{dP}{dt} = 3P$ ← implicit, linear vs.

$\frac{dP}{dt} = 3t$

$P = Ae^{3t}$ ← initial population, growth/death rate

$P = \frac{3}{2}t^2 + C$

$\frac{dP}{dt} = Ae^{3t} \cdot 3 = P \cdot 3$

$P = Ae^{3t}$ ← explicit (solution)

4. **Ex:** Verhulst Model

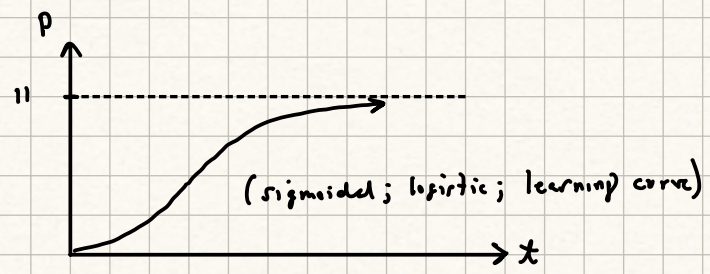
$$\frac{dP}{dt} = k\left(1 - \frac{P}{C}\right)P$$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{C}\right)$$

Suppose $K = 3$ and $C = 11$

$$\frac{dP}{dt} = 3P\left(1 - \frac{P}{11}\right)$$ ← implicit (non-linear)

$$P = \frac{11e^{3t}}{e^{3t} + D}$$ ← explicit



$$\frac{dP}{dt} = 3P\left(1 - \frac{P}{C}\right) + * \quad \longleftarrow \text{no solution (algebraically)}$$