

Practice on Basic Computations with Continuous Dynamical Systems

Mathematics 302—Chaos Theory

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Directions: Please complete the following exercises. You may confer with one another on strategy, but the spirit of the assignment is that you each learn the material and ultimately write up your own solutions.

1. Write p as an explicit function of t in the following continuous dynamical system (CDS): $p' = t^3 - 4t$, $p(0) = 2$.
2. Write p as an explicit function of t in the following continuous dynamical system (CDS): $p'' = t^3 - 4t$, $p(0) = 2$, $p'(0) = 4$.
3. According to Newton's Law of Gravity, the force exerted by gravity on an object of mass m near a more massive object of mass M is given by $F = \frac{mMG}{R^2}$, where R is the distance between the centers of mass of the objects, and G is the gravitation constant, which is about equal to $6.673 \times 10^{-11} \frac{Nm^2}{kg^2}$. The mass of the planet Mars is approximately $6.34 \times 10^{23} kg$, and its radius is about $3.43 \times 10^6 m$.
 - 3.1 Using the method outlined in class, determine the acceleration of an object due to gravity near the surface of Mars.
 - 3.2 Use the aforementioned acceleration to write the implicit form of a CDS that describes the motion of an object tossed from a height of $100m$ above the planet's surface with an initial velocity of $3 \frac{m}{s}$.
 - 3.3 Use your knowledge of antiderivatives to find a function which gives the object's velocity at any given time t .
 - 3.4 Use your knowledge of antiderivatives to find a function which gives the object's position at any given time t .
 - 3.5 Determine the time at which the object reaches the surface of Mars.
 - 3.6 Determine the velocity with which it reaches the surface.
4. For each of the following implicit CDSs, make an educated guess at which function or functions may be the explicit form, and then verify your guess.
 - 4.1 $y'' = 0$
 - 4.2 $y' = y$
 - 4.3 $(y')^2 + y^2 = 1$
 - 4.4 $y' + y = e^t$
 - 4.5 $y'' + y = 0$
 - 4.6 $ty' + y = 3t^2$