

**01:** If we do not understand precisely why a result holds, we may incorrectly extrapolate to a wrong conclusion.

**02:** Deeper knowledge is often needed to answer correctly questions posed in a simpler context.

## 2. Proper vs. Improper - (The Science Before the Science, Rizzi)

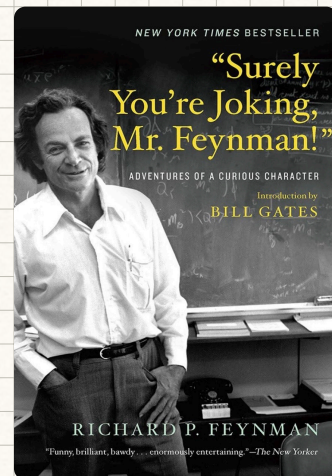
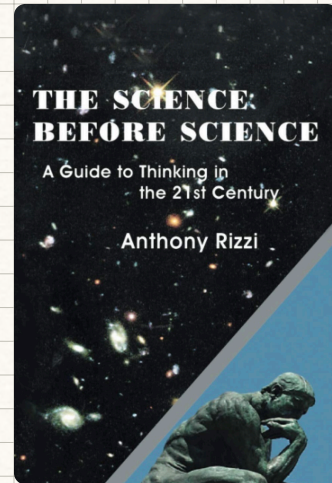
Science is a process, not a fixed body of knowledge.

This process requires constant scrutiny; dissenting voices are most valuable ones.

**Ex:** Newton's Law of Gravity  
Einstein's General Theory of Relativity

**Ex:** Alfred Wegener in early 1900s  
Geologist

**Ex:** Fr. George Lemaitre in 1927  
Astrophysicist



"Science is too important to be left to the experts."

In science, data rules, not degrees.

Consensus does not settle things; data does.

- Improper knowledge - knowledge from a trusted source; authority. [knowledge is necessarily conveyed.]
- Proper knowledge - knowledge apprehended for oneself, either by direct observation and experience or by logical thought.

"The task of the educator is to make the child's spirit pass again where its forefathers have gone, moving rapidly through certain stages but suppressing none of them. In this regard, the history of science must be our guide."

Henri Poincaré



1854-1912

Education is moving from improper to proper knowledge.

3. **Ex:** Sum of angles of a triangle is  $180^\circ$ .

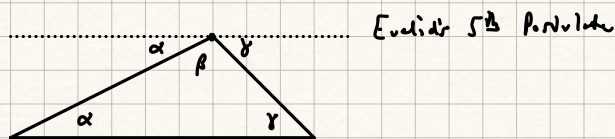
① Authority

② Conjecture (Empirically)

- Protractor
  - GSP; Geogebra
  - Death of Proof
- } Develops intuition but does not establish it for all cases.

③ Proof (Rigor)

- How do we test all cases if there are an infinite number of them?
- Abstraction - allows us to prove a result for an entire class of objects by considering an arbitrary example of one that is assumed to have only those properties that every object of the class possesses.



$$m\angle\alpha + m\angle\beta + m\angle\gamma = 180^\circ.$$