3. **Ex:** Sum of angles of a triangle is 180°.

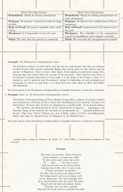
① Authority

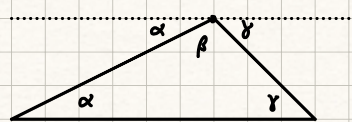
② Conjecture (Empirically)

- Protractor
- GSP ; Geogebra
- Death of Proof

Develops intuition but does not establish it for all cases.

③ Proof (Rigor)

- How do we test <u>all</u> cases if there are an infinite number of them?
- Abstraction – allows us to prove a result for an entire class of objects by considering an arbitrary example of one that is assumed to have only those properties that every object of the class possesses.
- 



Euclid's 5th Postulate

$$m\angle\alpha + m\angle\beta + m\angle\gamma = 180°.$$

- What was accomplished?
  ① Tested all cases.
  ② Gives insight into why the result is true.
  ③ Shows how result depends on other properties.

- **Suggestion**: encourage use of pictures, but only label what is given and only argue from what is labelled.

4. **Epicurus + Proclus** –

- Epicurus claimed obvious results need no proof. " Triangle inequality is obvious even to a donkey. "
- Proclus
  - Logical Link (why it's true; additional insight)
  - Training Thought Process (aid us in situations where results are not obvious)

5. **Rigor** – involves establishing a logical link between assumptions (axioms, postulates) and conclusions.

A good proof should:

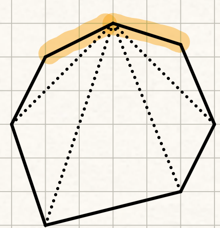
1. establish link
2. give clear insight into why result is true
3. apply to all cases of interest

Ex: Algebra Trick

$$\frac{3n+12}{3} = n+4$$

$$\frac{-n}{4}$$

Ex: Sum of interior angles of an n-gon is $(n-2)\cdot 180°$.



$n-2$ triangles

Ex: $\binom{n}{k} = \binom{n}{n-k}$ or $nCk = nC(n-k)$

$$\binom{13}{5} = \binom{13}{8}$$

↑

By choosing 5, we are choosing 8 to leave behind.

Ex: $2^{2^n}+1$ is prime for all $n \in \mathbb{N} \cup \{0\}$. (Mersenne Primes)

$n=0,\quad 2^{2^0}+1 = 2^1+1 = 3.$ ✓

$n=1,\quad 2^{2^1}+1 = 2^2+1 = 5.$ ✓

$n=2,\quad 2^{2^2}+1 = 2^4+1 = 17.$ ✓

$n=3,\quad 2^{2^3}+1 = 2^8+1 = 257.$ ✓

$n=4,\quad 2^{2^4}+1 = 2^{16}+1 = 65{,}537.$ ✓

$n=5,\quad 2^{2^5}+1 = 2^{32}+1 = 4{,}294{,}967{,}297$

$= 641 \cdot 6{,}700{,}417$ ✗

Ex: Any even integer ≥ 4 is the sum of 2 primes. (Goldbach Conjecture)

$4 = 2+2$

$6 = 3+3$

$8 = 3+5$

$10 = 3+7$

$12 = 5+7$

$\vdots$

$28 = 5+23 = 11+17$

$30 = 7+23$

$\vdots$

?

Ex: $x^n + y^n = z^n$ has no integer solutions if $n > 2$ $(n \in \mathbb{N})$

(If $n = 2$, Pythagorean Triples)

(Fermat's Last Theorem)

Search for Proof

Number Theory
- Encryption
- Coding Theory

Abstract Algebra
- Chemistry
- Physics / String Theory
- CAS
- Coding Theory

Combinatorics
- Genetics
- Probability
- Coding Theory

6. **Inductive vs. Deductive Reasoning** –