Axiomatic Systems

1. **Hvidsten** – Greek's pursuit of "harmony of proportion" and "perfection of reason."

2. **Axiomatic System** – a logical system founded upon a set of clearly articulated assumptions (axioms, postulates) and including all possible **ramifications** of those assumptions (known & unknown).

   ramifications = consequences

   Ex: Euclid's 5th Postulate has many equivalent form. The most common is Playfair's Postulate:

   "Given a line and a point not on the line, the exists exactly one line parallel to the given line and passing through the given point."

   Others: "The sum of the angles in a triangle is 180°."

3. **Aristotelean Logic** –

   - **Law of Excluded Middle**: $p \wedge \sim p$ is F. (p and its negation cannot be true at same time & in same way.)
   - Avoid Contradiction
   - cf. Fuzzy Logic – extends traditional logic to less exclusive situations – goal of more accurately reflecting every day language. Hoped it would advance AI.
   - cf. Eastern Philosophies (e.g. Buddhist koans, etc.); story w Ravi

4. **Elements of an Axiomatic System**

   ① **Consistency** – no inherent contradictions

   - **Importance** – avoid proving results about $\emptyset$.
   - **Establish** – build a model that satisfies all axioms simultaneously.

   ② **Independence** – none of the axioms can be proven from the others; no redundant axioms.

   - **Importance** –
     - minimal assumptions ⟶ widest applicability
     - clearest reasons for truth of theorems.
     - cf. Rule of Parsimony
   - **Establish** – replace axiom in question with its negation and check for consistency
     - If consistent (can build a model), then original axiom was independent of others.
     - If inconsistent, original was dependent on others.

③ **Completeness** - if all conjectures which can be formulated w/in the axiomatic system can be established as T or F w/in the axiomatic system. (No undecidable propositions.)

Ex: $\mathbb{Z}$ is not complete.

$3x - 12 = 0$

$x = 4$ ✓

$3x - 11 = 0$

$x = 11/3$ ✗

Ex: $\mathbb{Q}$ is not complete

$1x^2 - 2 = 0$

$x = \pm\sqrt{2} \in \mathbb{Q}$

Ex: $\mathbb{R}$ is not complete

$1x^2 + 1 = 0$

$x = \pm\sqrt{-1} = \pm i$

Ex: $\mathbb{C}$ is complete