

- Synthetic Geometry - based on constructions w/ a straight-edge and compass (divider). Purpose of a construction is to confine your thoughts to Euclidean approved thoughts - so it should lead to a proof.

- Analytic Geometry - René Descartes formulated a method for identifying any point in Euclidean space w/ an ordered pair of numbers \longrightarrow write algebraic formulas for geometric objects \longrightarrow solve geometry problems using algebra \longrightarrow Calculus and modern mathematics.

3. Euclid's 5th Postulate - is equivalent to:

- \exists at least one triangle \exists : the sum of its angles is congruent to 2 right angles.
- \exists at least one pair of similar, non-congruent triangles.
- A circle can be constructed through any 3 non-collinear points.
- \exists an upper limit to the size of a triangle.
- * • Given a line l and a point P not on l , $\exists!$ line through P and parallel to l .
(John Playfair, Scottish mathematician (1748 - 1819), but Proclus?)

4. Non-Euclidean Geometries -

- consistency \longleftrightarrow no contradictions \longleftrightarrow build a model
- independence \longleftrightarrow no redundancy, no axiom can be proven from the others \longleftrightarrow negate an axiom and build a model

S: Playfair Postulate

$\sim S$: Given a line l and a point P not on l ,

- ① there is no line through P and parallel to l , or
- ② there is more than one line through P and parallel to l .

Establish Consistency w/ $\sim S$: (Poincaré) Hyperbolic Geometry



