

Synthetic Geometry & Constructions

1. **Note** - Teaching Euclidean Geometry w/o constructions is like teaching Calculus w/o derivatives.

2. **Synthetic Geometry** - based on construction. Tools are fashioned from axioms w/ purpose:

- ① keep the geometer honest
- ② lead to proof

3. **Analytic Geometry** - linked to algebra by a coordinate system, allowing us to write algebraic equations for geometric objects.

4. **Euclidean Tools** - straight-edge and divider (no protractors or rulers).

The following are equivalent to Euclidean Tools:

- ① SE and Modern Compass
 - ② Compass alone (Mascheroni, 1822)
 - ③ SE and Rusty Compass
 - ★ ④ SE, Compass, and Parallel Line Constructor and Midpoint Constructor
- 2nd Order Euclidean Tools

5. **Basic Constructions** -

1. Basic Constructions

- (a) perpendicular bisector, given a segment
- (b) bisector, given an angle
- (c) copy an angle
- (d) midpoint, given a segment
- (e) perpendicular to a given line from a given point on the line
- (f) perpendicular to a given line from a point not on the line
- (g) parallel line, given a line and a point not on the line
- (h) tangent to a given point on a given circle
- (i) two circles tangent to one another, given their radii
- (j) two circles orthogonal to one another, given their radii
- (k) divide a given segment into n congruent pieces
- (l) tangent to a given circle, given a point not on the circle

2. Common Geometric Shapes

- (a) equilateral triangle
- (b) square inscribed in a circle
- (c) regular pentagon inscribed in a circle
- (d) regular hexagon inscribed in a circle
- (e) circle, given three points
- (f) a golden rectangle, given a unit segment
- (g) a golden spiral, given a unit segment

3. Constructible Numbers

- (a) $a \cdot b$, given segments of length a , b , and 1
- (b) $\frac{a}{b}$, given segments of length a , b , and 1
- (c) $a + b$, given segments of length a and b
- (d) $a - b$, given segments of length a and b
- (e) \sqrt{a} , given segments of length a , b , and 1