A Brief Introduction to the Nature of Reason

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Inductive Reasoning	Deductive Reasoning
Foundation: Makes no formal assumptions.	**Foundation:** Begins by making assumptions (axioms, postulates).
Purpose: To uncover a pattern in what is being studied.	**Purpose:** To discover the ramifications of the assumptions.
Role of Proof: No proof is possible; only verification.	**Role of Proof:** Logical proof of the discoveries is the goal.
Weakness: It is impossible to test all cases.	**Weakness:** The reliability of the assumptions cannot be established with complete certainty.
Faith: We trust that the pattern is consistent.	**Faith:** We trust that the assumptions are reliable.

The goal of reason, of course, is to uncover the truth as best we can. Each method of reason we employ has its strengths and its weaknesses. And, every method of reason that we employ has as its foundation *faith* of some kind; that is, there is a foundation upon which we put our trust. In inductive reasoning, this trust is in the orderliness of the universe. In deductive reasoning, we place our trust in the reliability of the assumptions that we make.

In practice, of course, both of these types of reason are used together. Often times, we use inductive reasoning to verify the reliability of the assumptions that we make in our deductive reasoning. Conversely, we often use deductive reasoning to help uncover the pattern that we seek in inductive reasoning. Let's consider a few examples.

1. **Example:** The Declaration of Independence states:

   > We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness.–That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed, –That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it, and to institute new Government, laying its foundation on such principles and organizing its powers in such form, as to them shall seem most likely to effect their Safety and Happiness ...

   Does this aspect of the Declaration of Independence exemplify inductive or deductive reasoning?

2. **Example:** Later on, the Declaration of Independence states:

   > The history of the present King of Great Britain [George III] is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States. To prove this, let Facts be submitted to a candid world. [A list of facts follows] ... We, therefore, the Representatives of the United States of America ... appealing to the Supreme Judge of the world for the rectitude of our intentions, do ... solemnly publish and declare, That these United Colonies are, and of Right ought to be Free and Independent States; that they are Absolved from all Allegiance to the British Crown ...

   Does this aspect of the Declaration of Independence exemplify inductive or deductive reasoning?

3. **Example:** All objects close to the earth, when in free fall in a vacuum, accelerate at $9.8\frac{m}{s^2}$ $\left(32\frac{ft}{s^2}\right)$. Obviously, to prove that this is universally true, we would have to test every object on earth, every grain of sand, every drop of water, at every time in history. This is clearly impossible. But, this theory has been tested over and over again in all parts of the world and by many generations of physicists. Hence, we *choose* to trust that the pattern is consistent. Our faith (trust) is so great, that we often call this a *proven fact*, though it truly is not a proven fact, since its truth has not been proven to be universally true. It is, however, a well-tested fact. Is this inductive or deductive reasoning?

4. **Example:** No, suppose we take the well-established theory that all objects close to the earth, when in free fall in a vacuum, accelerate at $9.8\frac{m}{s^2}$ $\left(32\frac{ft}{s^2}\right)$. Then, using Calculus, we deduce that, since $h'' = -9.8,\ h = -4.9t^2 + Ct + D$. Is this inductive or deductive reasoning?

We mathematicians are responsible for sometimes misleading our students about the nature of mathematics. Some enter the discipline because it is an area where there is "one right answer." While this is true with the deductive reasoning we use in mathematics, it is not true with the the inductive reasoning. When choosing our axioms, postulates, and definitions, there in fact isn't always one correct answer.

In addition, at its heart, mathematics is deductive. But, as we saw with Euclid's work, the ramifications of a few assumptions can be astronomical. The logical world that we create with those assumptions quickly becomes so large that we ourselves are lost in it. Thus, we often speak of "discovering" new theorems. It is for this reason that mathematicians must be highly creative, and there are a large number of mathematicians who are also musicians or artists of some sort. Mathematics is much like a treasure hunt. One of the wonderful aspects of mathematics is that it provides the thrill of seeking a pattern (much like science does), but without the limitations that come with inductive reasoning. In mathematics, one can actually *prove* that the pattern we find is real and universal, because mathematics is deductive in nature. Of course, we can never be absolutely sure that our results accurately reflect the real world; that is the limitation of mathematics.

I'll conclude with a poem by Clarence R. Wylie, Jr. (1911-1995), a mathematics professor at Furman University.

Paradox

Not truth, nor certainty. These I forswore
In my novitiate, as young men called
To holy orders must abjure the world.
"If . . . , then . . . ," this only I assert;
And my successes are but pretty chains
Linking twin doubts, for it is vain to ask
If what I postulate be justified,
Or what I prove possess the stamp of fact.
Yet bridges stand, and men no longer crawl
In two dimensions. And such triumphs stem
In no small measure from the power this game,
Played with the thrice-attenuated shades
Of things, has over their originals.
How frail the wand, but how profound the spell!