

# Project #1 (High School Geometry)

Mathematics 308—Modern Geometry

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**Directions:** What follows is a brief review of the major concepts and theorems of high school geometry.\*

- Read the following information.
- Then, using only S25, prove S5, S6.
- Then, using only S25 and S07, prove S10.
- Finally, using only those theorems relating to parallel lines and triangles, along with S21, to prove S11, S12, S18, S20, S22. It may help to prove S22 first, and then use it to prove S20 and S18 (this is not necessary, but it may be helpful).

## Selected Concepts from High School Geometry

1. An angle is the set of points on two rays with a common endpoint.
2. An acute angle has a measure between  $0^\circ$  and  $90^\circ$ , and an obtuse angle has a measure between  $90^\circ$  and  $180^\circ$ .
3. Two angles are complementary if the sum of the measures is  $90^\circ$ . Two angles are supplementary if the sum of the measures is  $180^\circ$ .
4. Three noncollinear points determine a triangle consisting of the three segments whose endpoints are the given points. Some types of triangles are:
  - *Equilateral*: All sides are congruent.
  - *Equiangular*: All angles are congruent.
  - *Isosecles*: At least two sides are congruent.
  - *Scalene*: No two sides are congruent.
  - *Acute*: All angles measure less than  $90^\circ$ .
  - *Right*: One angle measures  $90^\circ$ .
  - *Obtuse*: One angle is obtuse.
5. A circle consists of all points in a plane equidistant from a given point.
6. A polygon is a plane figure formed by the union of a finite number of line segments. The segments meet only at endpoints such that any two segments meet, at most at one point, and each segment meets exactly two other segments. Convex polygons have all interior angles less than  $180^\circ$ . Polygons are named according to the number of line segments composing the figures, as follows: Triangle (3), Quadrilateral (4), Pentagon (5), Hexagon (6), Heptagon (7), Octagon (8), Nonagon (9), Decagon (10).
7. Some types of quadrilaterals are:
  - *Square*: Four sides congruent, with four right angles.
  - *Rectangle*: Four right angles

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\*As found in *Modern Geometries, 5<sup>th</sup> Edition* by James R. Smart, 1998, Brooks/Cole Publishing Company, Pacific Grove, CA, pp. 409-411.

- *Rhombus*: Four equal sides.
  - *Parallelogram*: Opposite sides parallel.
  - *Trapezoid*: Exactly one pair of parallel sides.
8. The sum of the measures of the vertex angles in a convex polygon of  $n$  sides is  $(n - 2) \cdot 180^\circ$ . If the polygon is regular (all sides congruent and all angles congruent), then the measure of each vertex angle is  $\frac{(n - 2) \cdot 180^\circ}{n}$ .
  9. Polyhedra are three-dimensional figures whose faces are polygonal regions (polygons and their interiors). Prisms are polyhedra with two congruent opposite faces in parallel planes and corresponding edges connected with parallelograms. Antiprisms have two congruent opposite faces in parallel planes, but the edges of one face are connected to the vertices of the other, forming triangular regions for the other faces. Pyramids are polyhedra with a polygonal region for a base and triangular regions for the other faces. A pyramid is formed by connecting a point, not in the plane of the base, with each vertex of the base.
  10. A circular cylinder is a three-dimensional figure formed by two identical circles in parallel planes, along with the surface formed by line segments connecting corresponding points on the circles.
  11. A circular cone is a three-dimensional figure with a circular region for the base, along with the surface formed by joining line segments from every point on the circle to a common point not in the same plane.
  12. The volume of any prism/cylinder is the (area of the base)  $\times$  (height). If the shape is a pyramid, then the above formula is multiplied by  $\frac{1}{3}$ .
  13. A sphere is the set of all points in space the same distance from a given point.
  14. Skew lines are two lines in space that are neither parallel nor intersecting.

## Selected Theorems

- S01. If two triangles have two sides and the included angle of one congruent, respectively, to two sides and the included angle of the other, then the two triangles are congruent.
- S02. If two triangles have two angles and the included side of one congruent, respectively, to two angles and the included side of the other, then the two triangles are congruent.
- S03. If two triangles have the three sides of one congruent respectively, to the three sides of the other, then the two triangles are congruent.
- S04. The segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is equal to one-half the length of the third side.
- S05. The sum of the measures of the angles of a triangle is  $180^\circ$ .
- S06. The measure of an exterior angle of a triangle equals the sum of the measures of the two nonadjacent interior angles.
- S07. If two triangles have two angles of one congruent, respectively, to two angles of the other, then the triangles are similar.
- S08. If two triangles have an angle of one congruent to an angle of the other and the including sides proportional, then the triangles are similar.

- S09. If two triangles have their corresponding sides proportional, then they are similar.
- S10. The altitude to the hypotenuse of a right triangle forms two right triangles similar to each other and to the original triangle.
- S11. The two diagonals of a parallelogram bisect each other.
- S12. The median of a trapezoid has a length equal to half the sum of the lengths of the bases.
- S13. The opposite angles of a parallelogram are congruent.
- S14. The two diagonals of a rhombus are perpendicular.
- S15. The sum of the measures of the interior angles of a polygon of  $n$  sides is  $(n - 2) \cdot 180^\circ$ .
- S16. If two polygons are similar, then any two corresponding sides have the same ratio.
- S17. In a circle or in congruent circles, equal chords are equidistant from the center.
- S18. If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.
- S19. The line joining the centers of two intersecting circles is the perpendicular bisector of the common chord.
- S20. An angle inscribed in a semicircle is a right angle.
- S21. A central angle in a circle is measured by its intercepted arc.
- S22. An inscribed angle is measured by half its intercepted arc.
- S23. An angle formed by a tangent and a chord to a circle is measured by half its intercepted arc.
- S24. The ratio of the measure of a sector of a circle to the measure of area of the entire circle is equal to the ratio of the degrees in the angle of the sector to  $360^\circ$ .
- S25. If two parallel lines are cut by a transversal, then the alternate interior angles are congruent and the corresponding angles are congruent.
- S26. (The Pythagorean Theorem) For a right triangle, if  $a$  and  $b$  are the lengths of the sides of the right angle and  $c$  is the length of the opposite side (the hypotenuse), then  $c^2 = a^2 + b^2$ .