

Investigating Polyhedra Activity
Mathematics 308—Modern Geometry
Dr. Barry Peratt

Directions: Using Polydron Frames, complete the following activities. The purpose of this assignment is twofold:

- To give you hands-on experience in investigating the fascinating world of polyhedra.
- To give you first-hand experience in the application of van Hiele Levels of geometric understanding.

1 Notation and Terminology

- Polyhedron: is a closed, 3-D geometric object composed of planar polygons in such a way that no two adjacent polygons form a straight angle.
- Face: one of the planar polygons composing a polyhedron.
- Edge: a side shared by two adjacent faces of a polyhedron.
- Vertex: the intersection of three or more edges of a polyhedron.
- Defect: the difference between 360° and the sum of the face angle at a given vertex.
- Dihedral Angle: the angle formed by two adjacent faces of a polyhedron.
- Vertex Configuration: the type and order of faces meeting at a vertex. Common notation* is of the form $a.b.c\dots$, where a, b, c, \dots represent the number of sides on each face. For example, the vertex configuration of a cube is $4.4.4 = 4^3$. Additionally, a configuration of $3.3.4.4 = 3^2.4^2$ represents a vertex at which two adjacent quadrilaterals and two adjacent triangles meet, whereas $3.4.3.4 \neq 3^2.4^2$ represents a vertex at which two triangles and two quadrilaterals meeting in an alternating pattern.
- Edge Type: indicates which types of faces define an edge. For example, an edge of 4.3 consists of the intersection of a triangle and a quadrilateral.

2 Investigating Vertices Only

We will first investigate the types of possible “corners” that a polyhedron can have. We restrict our attention to polyhedra composed entirely of one kind of regular polygon.

1. How many different possibilities are there for a vertex surrounded only by equilateral triangles? Build each type of vertex and record its vertex configuration and defect in the table below. *Do not build the entire polyhedron at this point – just one vertex.*

*We are adopting the notation used by Andrew Hume of AT&T Bell Laboratories in Murray Hill, NJ, 07974 in his Computing Science Technical Report No. 130, *Exact Descriptions of Regular and Semi-Regular Polyhedra and Their Duals*, November 1986.

2. How many different possibilities are there for a vertex surrounded only by squares? Build each type and record its vertex configuration and defect in the table below.
3. How many different possibilities are there for a vertex surrounded only by regular pentagons? Build each type and record its vertex configuration and defect in the table below.
4. How many different possibilities are there for a vertex surrounded only by regular hexagons? Build each type and record its vertex configuration and defect in the table below.

3 Investigating Polyhedra

1. *Platonic or Regular Polyhedra:* Regular polyhedra are those in which: (1) all faces are regular polygons, (2) all vertex configurations are the same, and (3) all faces are the same. Create three polyhedra which satisfy these criteria, and record your results in the table below.
2. *Archimedean or Semi-Regular Polyhedra:* Semi-regular polyhedra are those in which: (1) all faces are regular polygons, (2) all vertex configurations are the same, (3) more than one type of face is allowed. Create four polyhedra which satisfy these criteria, and record your results in the table below. One of the polyhedra you create must have a vertex configuration of 3.4^3 .
3. *Irregular Polyhedra #1:* Suppose we still require all faces to be regular polygons and all faces to be the same, but we allow different vertex configurations. Build two such polyhedra, and record your results in the table below.
4. *Irregular Polyhedra #2:* Suppose we still require all faces to be regular polygons but allow more than one kind of face and different vertex configurations. Build two such polyhedra, and record your results in the table below.
5. *Finding Patterns:* Study the data you have collected on all of the polyhedra and formulate a conjecture about the relationship between the number of vertices (V), edges (E), and faces (F) for a polyhedron.

4 Dihedral Angles

1. Compute the dihedral angle for the Tetrahedron (3^3).
2. Compute the dihedral angle for the Icosahedron (3^5).
3. Compute the dihedral angle for the Dodecahedron (5^3).

