

Quizzes #3 and #4—Solutions  
Mathematics Education 308—Modern Geometry  
Dr. Peratt

**Directions:** Answer the following questions briefly but completely.

1. The author quotes J.L. Heilbron, who claims that “We owe geometry to the tax collector.” Explain what Heilbron meant by that statement.

**Answer:** In ancient Egypt, the tax collectors needed to re-survey the land annually after the flood of the Nile. These surveyors, called “rope-stretchers,” used  $3 - 4 - 5$  right triangles to help them divide land into plots.

2. According to the author, what is the main advantage over the Egyptians that the Greeks had because of their abstract understanding of geometry?

**Answer:** The Greeks had a method of creating classes of objects (a form of abstraction) and hence could prove results about entire classes of geometric entities, rather than simply relying on ad hoc knowledge of a few specific geometric objects.

3. Complete *one* of the following:

- (a) Tell when and where did Thales lived, and identify at least 3 major contributions he made to humanity.

**Answer:** He lives in Miletus around 624-548 B.C. His principle contributions were:

- He was the “first scientist,” because he believed the world to be a rational world where universal truths about natural phenomena could discovered by abstracting the world into ideal terms. This view contrasted with the view that the structure and nature of the world was constantly at the whim of gods.
- He formed a bridge between Egyptian and Greek culture, because he visited Egypt to study Egyptian mathematics. Purportedly, he computed the height of the pyramids by use of similar triangles and shadows.
- He posed the question, “What was matter composed of?,” and incorrectly but logically answered, “water.”
- He was the “first mathematician,” because he used deductive reasoning to prove mathematical results. He has 5 geometric theorems attributed to his name.

- (b) Tell when and where did Pythagoras lived, and identify at least 3 distinguishing characteristics of his school of thought.

**Answer:** Pythagoras lived in Samos around 580-500 B.C. He was a student of Thales, and the distinguishing characteristics of his school of thought were:

- The interconnection of mathematics, religion, art, and music.
- He coined the words philosophy (love of wisdom) and mathematics (that which is learned).

- “Numbers rule the universe” was the motto of the Pythagoreans, and the search for harmony and perfection, as reflected in proportions, was of utmost importance to them.
- Of course, the development of the Pythagorean Theorem.
- The used of deductive reasoning to prove mathematical statements true or false (based on assumed axioms).

4. Explain briefly the work of David Hilbert.

**Answer:** His *Grundlagen der Geometrie*, published in 1899, was a careful development of Euclidean Geometry based on 21 axioms. He was the leader of the *formalist* school of mathematics in the early 1900’s, which tried to lay down the development of all of mathematics from first principles.

5. What two logical traps are inherent in the development of any axiomatic system? How did the Greeks overcome these two challenges?

**Answer:** Since each result must be derived from previous results, one may easily develop a never-ending stream of prior statements. As well, definitions must be constructed using other terms, which also have definitions, etc. The danger here would be that of circular reasoning, where the definitions of terms all refer to one another.

To overcome the first, they instituted the concept of an axiom, or assumption, which would simply be assumed and would constitute the starting point from which all other claims were proven. The solution to the second problem involved allowing a set of undefined terms from which all other definitions were derived.