

Study Guide for Sections 1.1, 1.2, and 1.4  
Mathematics Education 308—Modern Geometry  
Dr. Peratt

**Directions:** Read Sections 1.1, 1.2 and 1.4 in your text, and answer the following questions based on the reading.

1. The author quotes J.L. Heilbron, who claims that “We owe geometry to the tax collector.” Explain what Heilbron meant by that statement.

**Answer:** In ancient Egypt, the tax collectors needed to re-survey the land annually after the flood of the Nile. These surveyors, called “rope-stretchers,” used  $3 - 4 - 5$  right triangles to help them divide land into plots.

2. According to the author, there were practical reasons for using geometric shapes such as rectangles and circles. In particular, he states that circular huts “provide maximum living space for the area they enclose.” What does he mean to say here? That circular huts provide maximum area for the area they enclose? That makes no sense; so, rephrase his statement so that it makes sense and is accurate.

**Answer:** A circle provides maximum living area with a minimum perimeter, and hence a minimum amount of material used.

3. The author states that the Egyptians had a more empirical understanding of geometry and the Greeks a more abstract understanding. Explain what the author means by this.

**Answer:** The Egyptians saw geometry as something pertaining only to concrete objects in the world. Hence, they did not distinguish between a physical object in the shape of a triangle and the concept of triangle itself. This was manifested, for example, in their failure to distinguish between approximations and exact values of numbers. By contrast, the Greek schools of Thales and Pythagoras, in the tradition of Plato, understood physical reality to be a mere shadow of a more perfect abstract reality which could only be grasped in the mind. Thus, an object in the shape of a triangle was not itself a triangle, but rather a physical, imperfect rendering of the concept (or “Platonic Idea”) called “triangle.”

4. According to the author, what is the main advantage over the Egyptians that the Greeks had because of their abstract understanding of geometry?

**Answer:** The Greeks had a method of creating classes of objects (a form of abstraction) and hence could prove results about entire classes of geometric entities using deductive reasoning, rather than simply relying on ad hoc knowledge of a few specific geometric objects.

5. Tell when and where did Thales lived, and identify at least 3 major contributions he made to humanity.

**Answer:** He lives in Miletus around 624-548 B.C. His principle contributions were:

- He was the “first scientist,” because he believed the world to be a rational world where universal truths about natural phenomena could be discovered by abstracting the world into ideal terms. This view contrasted with the view that the structure and nature of the world was constantly at the whim of gods.
- He formed a bridge between Egyptian and Greek culture, because he visited Egypt to study Egyptian mathematics. Purportedly, he computed the height of the pyramids by use of similar triangles and shadows.
- He posed the question, “What was matter composed of?,” and incorrectly but logically answered, “water.”
- He was the “first mathematician,” because he used deductive reasoning to prove mathematical results. He has 5 geometric theorems attributed to his name.

6. Tell when and where did Pythagoras live, and identify at least 3 distinguishing characteristics of his school of thought.

**Answer:** Pythagoras lived in Samos around 580-500 B.C. He was a student of Thales, and the distinguishing characteristics of his school of thought were:

- The interconnection of mathematics, religion, art, and music.
- He coined the words philosophy (love of wisdom) and mathematics (that which is learned).
- “Numbers rule the universe” was the motto of the Pythagoreans, and the search for harmony and perfection, as reflected in proportions, was of utmost importance to them.
- Of course, the development of the Pythagorean Theorem.
- The use of deductive reasoning to prove mathematical statements true or false (based on assumed axioms).

7. What was begun by the schools of Thales and Pythagoras but not perfected until Euclid in about 300 B.C.?

**Answer:** The development of the Axiomatic Method. Thales and Pythagoras technically used a combination of inductive and deductive reasoning in their proofs, but Euclid’s school perfected the development of a pure Axiomatic System.

8. What types of perfection did the ancient Greeks pursue that led to the development of the Axiomatic Method?

**Answer:** The pursuit of perfection in reasoning led them to the development of the Axiomatic System.

9. What two logical traps are inherent in the development of any axiomatic system?

**Answer:** Since each result must be derived from previous results, one may easily develop a never-ending stream of prior statements. As well, definitions must be constructed using other terms, which also have definitions, etc. The danger here would be that of circular reasoning, where the definitions of terms all refer to one another.

10. How did the Greeks overcome these two challenges?

**Answer:** To overcome the first, they instituted the concept of an axiom, or assumption, which would simply be assumed and would constitute the starting point from which all other claims were proven. The solution to the second problem involved allowing a set of undefined terms from which all other definitions were derived.

11. Draw an analogy between the relationship of theorems to axioms and the relationship of defined terms to undefined terms.

**Answer:** Just as theorems are derived from axioms, which are assumed as a starting point, so defined terms are defined in terms of undefined terms, which are also assumed and taken as a starting point.

12. List all of Euclid's assumptions (i.e. his postulates/axioms and his common notions).

**Answer:** *Euclid's Axioms:*

- (a) To draw a straight line from any point to any point.
- (b) To produce a finite straight line continuously in a straight line.
- (c) To describe a circle with any center and distance.
- (d) That all right angles are equal to one another.
- (e) That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles.

*Euclid's Common Notions:*

- (a) Things that are equal to the same thing are equal to one another.
- (b) If equals be added to equals, the wholes are equal.
- (c) If equals be subtracted from equals, the remainders are equal.
- (d) Things that coincide with one another are equal to one another.
- (e) The whole is greater than the part.

13. According to the author, the great insight among the Greeks concerning an abstract axiomatic system was what?

**Answer:** Their great insight was that it was the relationships among objects and how we apply those relationships that matter, and not the objects themselves. For this reason, Plato claimed that "two" was the only number without meaning. If you have one, he reasoned, then you have one. But if you have two, you immediately have three, because you have the two *and* the relationship between those two, which makes three.

14. Briefly describe the point/counter-point dialogue between Epicurus and Proclus concerning Euclid's work.

**Answer:** Epicurus claimed that Euclid's careful axiomatic treatment of even the simpler results from Geometry was needlessly laborious since many of them were "obvious." Proclus claimed that, while some of the results were indeed obvious, understanding why they logically followed from axioms helps to train our mind to reason about things that we cannot perceive.

15. If you have seen the film, “Karate Kid,” the martial arts instructor makes the kid perform everyday chores with exhausting exactness and repetition (such as waxing a car or painting a fence). This is a common training technique in the martial arts such as T’ai Chi and some forms of Kung Fu. What was the point of this exercise in the movie, and how does this relate to the argument of Proclus in favor of Euclid’s work?

**Answer:** Many everyday chores could be done without much thought, but performing them with an exhausting exactness and repetition helps to train the body to act in a disciplined and quick manner in those situations requiring more elaborate set of motions. This is essentially a physical analog of Proclus’ argument about the intellectual endeavors of Euclid – that they train the mind to think clearly and logically so that it might perform well when confronted with claims that are *not* obvious.

16. Explain briefly the work of David Hilbert.

**Answer:** His *Grundlagen der Geometrie*, published in 1899, was a careful development of Euclidean Geometry based on 21 axioms. He was the leader of the *formalist* school of mathematics in the early 1900’s, which tried to lay down the development of all of mathematics from first principles.

17. Complete Exercise 1.4.1 on page 23 of your text.

**Answer:** Using Random House Webster’s Unabridged Dictionary, I get

- power → ability → power.
- straight → (no) bend → curved → bent.
- real → true → conforming to reality.