

Quizzes #7, #8, & #9—Solutions
Reading Guide for Handout on Non-Euclidean Geometries
Mathematics 308—Modern Geometry
Dr. Peratt

Directions: Read the Handout on Non-Euclidean Geometries, and answer the following questions based on the reading.

1. Hilbert's rigorous axiomatization of classical Euclidean Geometry utilized 16 axioms in lieu of Euclid's 5 axioms. Into which categories were these axioms divided?

Answer: Betweenness, Congruence, Continuity, and Parallelism.

2. Name the three types of geometries that are based on variations of Euclid's Parallel Postulate.

Answer: Planar, Elliptic, Hyperbolic (I would have also accepted any of these: Projective, Finite, Absolute, Fractal).

3. Identify the main advantages and disadvantages of the Klein-Beltrami and Poincare' models of Hyperbolic Geometry.

Answer: For Klein-Beltrami, the notion of distance and angle are not intuitive. For Poincare', the notion of distance is not intuitive., but the traditional notion of angle in Euclidean Geometry is preserved. Both have the advantage of being a model in Euclidean Geometry for which there are many lines parallel to a given line through a given point.

4. What is a Lambert Quadrilateral, and why is it significant?

Answer: It is a quadrilateral in Hyperbolic Geometry with three right angles but whose interior angles do not add to 360° . It is important, because it shows that some basic properties of geometric shapes in Euclidean Geometry are not true in Hyperbolic Geometry.

5. What two properties does Projective Finite Geometry share with Elliptic Geometry?

Answer:

- Incidence
- Lack of Parallelism

6. Describe briefly the notion of *duality* for Projective Planes.

Answer: Since the axioms regarding points and lines are symmetric, any theorem in this geometry that is proven will automatically give a dual theorem that is also true – i.e. a theorem in which the notions of line and plane are reversed.

7. How do Projective Planes give rise to Affine Planes?

Answer: By deleting a single line and all of the points on that line. This gives rise to a situation in which Playfair's version of Euclid's Fifth Postulate holds.

8. Describe the main advantage of minimal geometries; that is, what advantage is there to having a minimal amount of structure in a geometry?

Answer: Any result proven in this geometry must necessarily hold in more complex geometries; hence, we can determine what properties the more complex geometries must all share because they arise from a minimal set of axioms.

9. What is Synthetic Geometry?

Answer: Construction-based geometry in which the tools used follow from the axioms in that geometry and for which constructions provide the outline for proofs.

10. Who is primarily responsible for the development of Analytic Geometry, and what was so powerful about this new type of mathematics?

Answer: Rene Descartes, and it is so powerful because it allows us to write algebraic equations for geometric objects, thereby making available all of the machinery of algebra in order to solve geometric problems.

11. Just as three different geometries can be formed by varying Euclid's Parallel Postulate, a number of non-Euclidean Geometries can be created by altering Hilbert's Axioms of Euclidean Geometry. Which four classes of geometries are created in this way?

Answer:

- Incidence Geometry
- Affine Geometry
- Projective Geometry
- Ordered Geometry

12. Fractal Geometry exists within Euclidean Geometry (and alters none of the axioms of Euclidean Geometry). So, what does Fractal Geometry include that classical Euclidean Geometry does not?

Answer: Additional analytic tools not available in traditional Euclidean Geometry, most especially the notion that the dimension of an object can be a non-integer value.

13. Write a short paragraph, after watching the Veritasium video on Non-Euclidean Geometries, the highlights one aspect of this story that you found particularly enlightening (mind-blowing or fascinating or otherwise interesting).