

Test #1, Part #1—Solutions
 Mathematics 308—Modern Geometry
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Directions: On a separate sheet of paper, neatly answer the following questions.

1. **Inductive vs. Deductive Reasoning:** List the five main differences between inductive and deductive reasoning.

Answer:

INDUCTIVE REASONING	DEDUCTIVE REASONING
Foundation: Makes no formal assumptions.	Foundation: Begins by making assumptions (axioms, postulates).
Purpose: To uncover a pattern in what is being studied.	Purpose: To discover the ramifications of the assumptions.
Role of Proof: No proof is possible; only verification.	Role of Proof: Logical proof of the discoveries is the goal.
Weakness: It is impossible to test all cases.	Weakness: The reliability of the assumptions cannot be established with complete certainty.
Faith: We trust that the pattern is consistent.	Faith: We trust that the assumptions are reliable.

2. **Short Answers:**

- (a) “Education involves moving from improper to proper knowledge,” says Anthony Rizzi in his book, *The Science Before the Science*. Define *proper* and *improper* knowledge.

Answer: Improper knowledge is gained from others and relies on their trustworthiness. Proper knowledge is gained for oneself through one’s senses or rational thought process.

- (b) In the section of the class notes entitled, *What Is Rigor?*, we identified three elements that a proof should possess. List them.

Answer: Rigor involves establishing a logical link between assumptions (axioms, postulates) and conclusions (theorems, etc.). Ideally, it should:

- Establish a logical link between assumptions and the result in question.
- Prove insight into why the result must be true.
- Apply to all cases of interest.

- (c) Fill in the missing words in this quote from the MN BOT Standards for Mathematics Education: “Students need to understand that while what are verified by observation, they are established as universally true only by recourse to what.”

Answer: empirical facts, formal proof.

- (d) The discovery of what knowledge in what library in what place led to a revival of ancient knowledge in Europe known as what?

Answer: Ancient Greco-Roman; La Alhambra; Granada (Spain); Renaissance.

- (e) In developing methods for tracking the movements of the stars and other celestial bodies, the ancient Babylonians developed a rudimentary understanding for what we know today to be what mathematical discipline?

Answer: Trigonometry.

- (f) In contrast to the Egyptians, who viewed mathematical knowledge as sacred and belonging only to a high priestly class, the Greeks promoted what, which led to a rich and varied body of knowledge about philosophy, mathematics, and the natural world?

Answer: “Democracy of Knowledge” or “Free Market Intellectualism.”

- (g) The role of mathematics is not to establish the truth value of specific statements at they relate to the real world. Rather, mathematics concerns itself with the truth value of what?

Answer: The truth value of the implication $p \rightarrow q$. We care not about the truth value of p nor of q , only whether $p \rightarrow q$ is a true implication.

- (h) The re-discovery of the writings of what ancient Greek philosopher caused intellectuals in medieval times to argue against the science of the High Middle Ages and Christianity in general, causing a monk by the name of Thomas Aquinas to write a book in which he argued that the two forms of knowledge were compatible?

Answer: Aristotle.

3. Short Paragraph Answers:

- (a) To illustrate the need for rigor, identify a conjecture that defies our intuition but has recently been proven or, with the advent of technology, has been disproven.

Answer: One example cited in class is the claim that $2^{2^n} + 1$ is prime for every natural number n . This is true for $n = 0, 1, \dots, 4$, but until the advent of computers, it was not possible to check the result for $n = 5$. In fact, $2^{2^5} + 1 = 4,294,967,297 = 641 \times 6,700,417$. Another possibility is Fermat's Last Theorem, which states that $x^n + y^n = z^n$ has no integer solution if n is an integer greater than 2; this was finally proven by Andrew Wiles of Princeton University in the early 1980's. Finally, we watched a video exploring the conjecture that $n^{17} + 9$ and $(n + 1)^{17} + 9$ are relatively prime; the pattern holds for every case up to $n = 842, 443, 292, 559, 288, 932, 928, 819, 732, 230, 890, 067, 249, 420, 460, 792, 400$ (I would not, of course, require you to remember that number!).

- (b) Briefly describe the role of the Muslims in the history of geometry. (Be sure to include how their conquest of much of the ancient world and the dictates of their own religion influenced their role.)

Answer: When they conquered parts of the Roman Empire, they inherited, preserved, and enriched the geometric knowledge of the Greeks and Romans, particularly through their artwork involving tessellations of the plane. When some of this land was conquered in the Middle Ages by the northern Europeans, this preserved and enhanced knowledge was brought back to Europe, prompting an era of great learning known as "The Renaissance."

- (c) Briefly explain how to argue from a picture in a way that guarantees the logical validity of your argument.

Answer: Label on the picture *only* things that are given and known. Then, when referencing the picture in your proof, make sure to reference only what is labeled in your picture.

4. Short Paragraph Answers with Choice:

- (a) For *two* (2) of the following contributors to geometry, briefly (in a sentence or two) describe their main contribution(s).

- Thales (624-547 B.C.)

Answer: For full credit, you needed to mention that Thales was the first to conclude that the universe was governed by natural laws and was not at the whim of the gods. You also could have noted that he founded the Ionian School in Miletus, proved 5 famous geometric theorems, and was the first to ask what matter was composed of.

- Hippocrates (460-375 B.C.)

Answer: Applied the idea of Thales, that the world does not operate at the whim of arbitrary and capricious gods but rather according to universal truths or laws, to the field of medicine. He maintained that illness had natural causes that could be discovered and that, therefore, illness could be treated.

- Archimedes (287-212 B.C.)

Answer: Archimedes was the engineer of the ancient Greeks and formulated the laws governing levers and pulleys and buoyancy, and he invented engines of war and the water screw.

- Eratosthenes (274-194 B.C.)

Answer: Eratosthenes was the director of the library in Alexandria and is best known for computing very accurately the circumference of the earth using only shadows and geometry.

- (b) For three (3) of the following ideas, events, places, or people, briefly (in a sentence or two) describe their significance and/or contribution in the historical development of geometry (or mathematics in general).

- Rope Stretchers

Answer: “We owe geometry to the tax collector,” because ancient Egyptian tax collectors needed to survey the land each year after the Nile flooded to clearly mark each person’s land ownership. To do this, they employed rope stretchers, who knew that a triangle with sides of length 3, 4, and 5 would necessarily make a right angle.

- Ionian School in Miletus

Answer: Thales founded the Ionian School in Miletus. He was the first to conclude that the universe was governed by natural laws and was not at the whim of the gods. You also could have noted that he founded the Ionian School in Miletus, proved 5 famous geometric theorems, and was the first to ask what matter was composed of.

- Pythagorean School in Samos

Answer: Coined the term philosophy (love of wisdom) and mathematics (that which is learned) and held that mathematics, religion, art, and music are interconnected. They also killed Hippasus for discovering the existence of irrational numbers and sharing the secret knowledge of the Pythagoreans with the general public.

- Aristotelean vs. Platonic thought

Answer: Plato and Pythagoras believed that the only way to discover truth was through the mind, and that which we perceive with our senses is but an imperfect rendering of the reality of Ideals, which can only be accessed through rational thought and reflection. Aristotle, a student of Plato, disagreed. He maintained that universal truths, Essences, could be found by carefully observing the physical world.