

Test #1, Part #2  
Mathematics 308—Modern Geometry  
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February 4, 2026

**Directions:** On a separate sheet of paper, neatly answer the following questions.

1. **van Hiele Levels:**

- (a) State each van Hiele level and briefly (in one sentence) describe its main characteristics.
- (b) Give a concrete example of reasoning at van Hiele level 1 (assuming they are numbered 0 through 4).

2. **Axiomatic Systems:** The three important properties of an axiomatic system are consistency, independence, and completeness. For each of these terms, define it and explain briefly how one can establish that a particular axiomatic system possess it.

3. **Short Answers:**

- (a) Aristotelian Logic is characterized by what “law” that distinguishes it from other types of logic, such as Fuzzy Logic and Eastern Mystical thought?
- (b) Cite the main argument against heliocentrism that was advanced by Aristotle and which Galileo could not counter (and in fact was not definitively countered until 1838).
- (c) Briefly describe the difference between a corollary and a lemma.
- (d) Briefly describe the great insight among the Greeks concerning abstract axiomatic systems identified by Hvidsten (the author of your text).
- (e) Identify the two logical traps that are inherent in the development of any axiomatic system and the way in which the Greeks overcame them.
- (f) Epicurus maintained that proving some of the more elementary results of Euclid was a waste of time, since those results are obvious. He famously quipped that “even a donkey knows to take the hypotenuse of a triangle to get to a stack of hay.” Proclus responded by noting that proving such “obvious” truths serves two purposes. List them.

4. **Short Paragraph Answers with Choice:**

- (a) Answer *one* (1) of the following:
  - Describe the main contribution of Bertrand Russell (1872-1970) to set theory. Be specific. Your explanation should include the phrases “set of all sets that are not elements of themselves” and “self-referential.”
  - Describe the main contribution of Kurt Gödel (1902-1978) to modern mathematics. Be specific. Your explanation should include the phrases “if  $P$  is consistent, then. . .” and “if  $P$  is not consistent, then. . .”
- (b) Answer *one* (1) of the following:
  - George Sim Johnston observed that “it was not simply a new theory of the nature of celestial movements that was feared, but a new theory of the nature of theory.” Explain briefly what he meant by that. Your explanation should include the phrase “saving the appearances.”
  - Timothy O’Neill identified the Renaissance a “curiously conservative and rather retrograde movement in many ways.” Explain briefly what he meant by that, and how it relates to Galileo’s main criticism of his contemporary scientists that caused him to label them “little Aristotles.”