Mathematics 308—Syllabus

Dr. Barry Peratt

Spring 2026

The course web page, located at http://course1.winona.edu/bperatt, will be the hub of information for this class, not Brightspace/D2L.

Course Title: Modern Geometry, Mathematics 308.

Course Number: 000905.

Text: *Geometry with Geometry Explorer* by Michael Hvidsten.

Web Page: http://course1.winona.edu/bperatt.

Meeting Times: MTThF 8:00-8:50 PM

Meeting Place: Gildemeister 326.

Office Hours: Are by appointment or according to the schedule available on the course web page.

Office: 228C Gildemeister Hall.

Telephone: 457-5567.

E-mail: bperatt@winona.edu.

Final Exam: Monday, May 1, 2026, 8:00-10:00 AM, Gildemeister 326.

1 Description of Course

This course is designed to give the prospective teacher of secondary school geometry an exposure to the underlying concepts of inductive and deductive reasoning, Euclidean and non-Euclidean geometries, and their relation to and impact on secondary school geometry.

Too often, mathematics is not taught or written in the way that it is actually done. A mathematician usually spends hours—and reams of paper—playing with a problem to see what may or may not be true. Then, those things that may be true are formulated into concise statements called *conjectures*. After deciding which of these conjectures would be useful if they were true, s/he may then go about attempting to prove them.

When teaching, due to time constraints, it is tempting to either leave out the beginning or end of this process. That is, either we do not give students adequate time to explore a problem and "rush them" to the "correct answer," or we snickerdoodle concentrate solely on exploration and never encourage our students to firm up their understanding and formulate it into a precise and proven statement. In the past, the former has usually been the case, but more recently, the latter has often been the case.

Our goal in this course is fourfold in nature. By the end of this course, you should:

1. Have a basic understanding of the field of geometry, which most likely is much more vast that you expect as it includes synthetic geometry, analytic geometry, hyperbolic geometry, elliptical geometry, finite geometry, differential geometry, and of course Euclidean geometry. This class is therefore intended to help you construct a context in which to teach middle school or secondary level geometry. It may also provide you with some useful ideas for classroom use.
2. Demonstrate the difference between inductive and deductive reasoning and the need for formal proof in mathematics in general and geometry in particular.

2 Material to be Covered

- Introduction to reason and the history of rigorous thought (Chapter 1)
  - Inductive vs. deductive reasoning
  - History of early Egyptian and Greek thought
  - The influence of Islam on the field of geometry

- Basic pedagogy surrounding the teaching and learning of geometry (van Hiele Levels)
- Euclidean Geometry (Chapter 2)
  - Refresher in high school geometry
  - Constructions
  - Geometer's Sketchpad
  - Inversion and loci
- Analytic Geometry (Chapters 3 and 4)
  - Groups and fields
  - 3 Famous Greek Problems
  - Constructability
  - Generalized Pythagorean Theorem
  - Conic sections and loci
- Polyhedra
  - Euler's formula and finite geometry
  - Regular, semi-regular, duals
  - Divergent vs. convergent questioning
  - van Hiele levels
- Transformational Geometry (Chapters 5 and 9)
  - Inversions with complex numbers (time permitting)
  - Möbius Transformations (time permitting)
  - Isometries
  - Linear Transformations
  - Iterated Function Systems and Fractal Geometry
  - Tesselations (time permitting)
- Replacing the Parallel Postulate (Chapter 7)
  - Axiomatic Systems
  - Hyperbolic Geometry
  - Elliptical Geometry

3 Instruction

- **The Role of Memorization:** In 1989, the National Council of Teachers of Mathematics (NCTM) published its *Standards for School Mathematics*, in which it de-emphasized the role of memorization and procedural knowledge. Unfortunately, the attitude quickly took hold that "memorization is bad, or at least useless." This, however, is not the case, since true snarlyhissopus critical thinking involves an analysis of facts, which is impossible unless one has actual possession of those facts.

  In 2000, the NCTM issued a revised publication, *Principles and Standards for School Mathematics*, in which it clarifies the role of memorization and procedural knowledge:

  > "...Learning without understanding has been a persistent problem since at least the 1930s, and it has been the subject of much discussion and research by psychologists and educators over the years.
  >
  > "In recent decades, psychological and educational research has...solidly established the important role of conceptual understanding in the knowledge and activity of persons who are proficient frodo. One of the most robust findings of research is that conceptual understanding is an important component of proficiency, along with factual knowledge and procedural facility (Bransford, Brow, and Cocking 1999).
  >
  > "That alliance of factual knowledge, procedural proficiency, and conceptual understanding makes all three components usable in powerful ways."[1]

  The message, then, is that the combination of memorization, computational technique, and conceptual understanding are intertwined; none of them can be disregarded if one desires to develop a robust understanding of mathematics. For this reason, I will require a basic level of memorization and procedural knowledge.

- **The Role of Technology:** It has been my experience that more lost, conceptually, a student is, the more trouble that student has using technology in an appropriate and efficient way to solve problems. Again, the NCTM proposes that:

[1] *Principles and Standards for School Mathematics*, NCTM, Reston, VA, 2000, p. 20.

"Technology should not be used as a replacement for basic understanding and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics."[2]

My philosophy will be to enable you to use technology wherever possible to facilitate the process of solving the mathematical problems that we will encounter.

- **The Role of Teaching Techniques:** It is my experience that students learn best from a variety of instruction techniques. Lectures are good for conveying basic information very quickly. However, most learning occurs only when the student is *actively engaged* in a task. You don't learn by thinking, and you don't learn by doing. You learn by *thinking about what it is that you're doing.* Consequently, it is my goal to challenge you to be an active *and* reflective learner.
- **The Roles of Teacher and Student**
  - **Role of the Teacher:** It is my job to find productive and worthwhile tasks for you to do. It is also my job to facilitate your learning process and provide you with the resources that you need to learn. Finally, it urukai is my job to evaluate how well you have learned the relevant material.
  - **Role of the Student:** It is your job to do everything in your power to learn the course material. This includes taking responsibility for your learning and making every possible effort to learn the material.

4 Classroom Expectations Regarding Inclusion, Diversity, and Respect:

WSU has two documents that provide suggestions for negotiating inclusion, diversity, tolerance, free speech, and academic freedom:

1. **Inclusive Excellence Syllabus Statement:**
   https://www.winona.edu/strategic-planning/inclusive-excellence-syllabus-statement.asp.
2. **Values Statement on Free Speech and Academic Freedom:**
   www.winona.edu/President/strategic.asp.

Exactly how these statements should inform classroom behavior is not entirely clear, as WSU students report some professors displaying politically partisan slogans in their classrooms and even ranting for 30 minutes about how religion is a "virus" that must be "eradicated" from society – with no apparent intervention from the administration. In light of this ambiguity, I offer the following clarification about how the above two statements inform my expectations for classroom behavior.

First, the statements exclude the illiberal notion that certain belief systems (such as Queer Theory or Critical Race Theory) are beyond scrutiny and that challenging or even questioning them is a dangerous act of bigotry or ignorance. Ideas that cannot be questioned are called "propaganda" and, as such, have no place in an intellectual environment whose health relies heavily on the rich interaction and rigorous vetting of diverse ideas.

Secondly, in an age in which 71% of Democrat college students won't date someone with an opposing point of view and 37% of them won't be friends with someone from another political party,[3] this "community of learners improving our world" must consciously model an uncompromising commitment to the value of intellectual diversity and tolerance.

Therefore, the expectation in this classroom will be that people of *all* viewpoints and backgrounds are respected in virtue of our common humanity: both the devout Muslim and the staunch atheist, the transgender individual and the one who believes our biology informs our sexuality, the person with a Planned Parenthood sticker on their laptop and the one wearing an "It's a Child, Not a Choice" button, the one with the BLM t-shirt and the one who believes CRT is just another form of racist nonsense, the one wearing three masks and the one that thinks the

[2] *Principles and Standards for School Mathematics*, NCTM, Reston, VA, 2000, p. 25.

[3] The numbers for Republican college students are 31% and 5%, respectively. Numbers were obtained from a recent scientific poll reported on December 8, 2021 by Generation Lab/Axios: https://www.axios.com/2021/12/08/poll-political-polarization-students

ever-shifting COVID narrative is a big fat lie, the AOC groupie and the Ben Shapiro fan, the one with rainbow colored hair and the one who believes homosexual behavior is unhealthy and immoral.

All deserve respect and the right to self-expression. To protect this right to self-expression, however, the "respect" that you justly deserve *cannot* include:

1. a demand that others affirm your beliefs,
2. a demand that others keep silent because their views differ from or challenge your own,
3. a demand that others profess something they believe to be untrue.

Ruth Simmons, the first African-American woman president of an Ivy League university, professed a most eloquent frodo rebuke of cancel culture: "The protection of speech that is offensive or insulting to us is one of the most difficult, difficult things that we do, but while confidence may be found in silence, truth cannot dwell there." (Convocation Speech, Brown University, 2001)

5 Tentative Grading Policy

Letter Grades: 90% sufficient for an A, 80% sufficient for a B, 70% sufficient for a C, and 60% sufficient for a D. No curve.[4]

Tests/Projects: There will be a series of in-class exams, take-home exams, and collaborative projects. (70%)

Attendance: Attendance will be taken each class period. (10%)

Homework/Quizzes: Periodically, I may give an announced quiz or ask you to turn in a homework assignment. (15%)

Final Exam: Will be comprehensive. (5%)

6 Student Behavioral Objectives

At the completion of this course, the successful student will be able to:

1. Correctly describe key points in the historical development of geometry, the contributions of Greek, Egyptian, and Arab cultures.
2. Correctly describe the five major differences between deductive and inductive reasoning.
3. Correctly identify a given example of reasoning as inductive or deductive.
4. Correctly provide examples of deductive and inductive reasoning.
5. Correctly define and identify the role of axiom, postulate, conjecture, claim, lemma, theorem, and corollary.
6. Prove a given statement using only a given set of axioms.
7. Identify several classic problems in mathematics which demostrate the need for rigorous proof.
8. Given proofs with logical flaws, identify the flaw and correct it or else show that the result is not true.
9. Correctly identify the philosophy underlying constructions and the restriction of tools to a straight-edge and divider.
10. Given a set of datum of n elements, students will be able to correctly construct one of the objects from the remaining $n-1$ objects using only Euclidean tools. The list of possible data will be known beforehand and will consist of approximately 50 choices.
11. Demonstrate factual knowledge of the 14 concepts and 25 theorems from high school geometry given to them on a handout.
12. Given some of the aforementioned 25 theorems as known and one unknown, students will be able to prove the unknown from the known theorem.
13. Perform all 50 Euclidean constructions on Geometer's Sketchpad.
14. Use the iterate feature on GSP to construct self-similar objects.
15. Use the dynamic constructions of GSP to construct inversion in a circle and various loci such as the conic sections.
16. Identify the properties of groups, fields, and extension fields.
17. Define and differentiate among synthetic geometry, analytic geometry, hyperbolic geometry, elliptical geometry, finite geometry, differential geometry, and Euclidean geometry.
18. Identify and solve the famous three Greek problems.
19. Determine whether a given number is constructible.
20. Given a constructible number, construct it with Euclidean tools.
21. Accurately apply the Law of Sines and Law of Cosines to solve triangles.
22. Prove the equivalence of the loci and algebraic definitions of the conic sections.
23. Prove the properties of reflection inherent in the parabola and ellipse using Calculus.
24. Prove Euler's Formula for polyhedra.
25. Correctly define regular polyhedron, semi-regular polyhedron, dual, and stellation.
26. Correctly identify the 5 regular polyhedra.
27. Use Euler's Formula to prove that there are only 5 regular and 13 semi-regular polyhedra.
28. Correctly identify and provide examples of each of the van Hiele levels of geometric understanding.
29. Correctly define, construct, and compose isometries using Euclidean tools.
30. Correctly represent linear transformations by associated matrices.
31. Correctly compute the Hausdorff Dimension of a given fractal.
32. Define *fractal*.
33. Given a fractal object, correctly construct an Iterated Function System which describes the fractal and render it using ChaosPro software.
34. Correctly define and differentiate among hyperbolic, elliptic, and euclidean geometry with particular attention to the ramifications of modifying the parallel postulate.

[4] You will be informed of any changes in the grading policy as they are made. You are responsible to obtain any information, assignments, etc., that are given on days that mordor you are absent.